Information Economies with Taste Diversity and Bounded Attention Spans[1]

John P. Conley[2]

Vanderbilt University

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Abstract

We consider an economy with a countably infinite number of consumers and pure public information goods. Each of these differentiated products is produced by a single monopoly firm that enters the market if it can cover costs. Thus, the product space is endogenous. We assume that the population of agents has diverse tastes, and also bounded attention spans for content. We show that this implies that at all Pareto efficient allocations, all agents consume a finite number of public goods, and that each public good is consumed by a finite number of agents, In effect, these two taste assumptions turn pure public goods into what amount to club goods, despite the lack of rivalry in consumption or crowding of any type. Unfortunately, the equilibrium outcomes of Tiebout-like competition between public good providers do not satisfy the First Welfare Theorem. Even nonanonymous Lindahlian price systems are not sufficient to signal all profit opportunities to firms. We conclude that information markets are likely to be inefficient, and there will always remain opportunities for economic profits in an information economy.

Keywords: countably infinite economy, public goods, Tiebout, local public goods. Internet economics, ICT economics, information economics, competitive equilibrium, general equilibrium, innovation, product differentiation, price discrimination.

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2 Department of Economics, Vanderbilt University, Nashville, TN 37235, j.p.conley@vanderbilt.edu

1. Introduction

Modern communication technologies have immeasurably expanded the addressable market for creative works, social media platforms, and other information services. As number and variety of information goods has proliferated, we see ordinary people spending ever increasing parts of their income, and especially their time, engaging with and consuming them.

Information goods and services have always had high first copy costs and relatively low marginal costs. Marginal costs typically remain below average costs until the quantity produced gets very large. Since information goods are not perfect substitutes, each producer can therefore be seen as kind of local, natural monopolist, providing a unique differentiated product.

Until recently, information goods were produced as physical media, such as books, records, and films. Distribution required a real-world logistic, wholesale, and retail network. While the unit costs of serving each consumer were low compared to first copy costs, they were still non-trivial. Physical media can be shared, but it degrades as it is used or copied, and there is a degree of rivalry in passing a book or record back and forth.

Electronic information goods, on the other hand, can be consumed by any number of agents without rivalry, without significant degradation, and at little additional cost to producers. Thus, modern information goods are closer to being nonrival public goods with zero unit costs.

New books, recordings, blog entries, poems, patents, videos, and so on, are produced by many kinds of creators. Electronic distribution has made entry into information markets less costly, but producers still must compete with one another to produce content that finds a sufficient audience to cover costs.

Putting this together, we argue that the production side of an information economy has the following features:

- Each good is produced by a natural monopoly, and is a discrete item, rather than a quantity of a divisible homogenous good.
- There is free entry to markets for firms wishing to offer new, differentiated, information goods.
- Information goods are close to non-rival in consumption.
- The information product space in endogenous in the sense that firms enter and produce their goods only if there exists sufficient willingness to pay across consumers to cover costs.

In other words, the supply side can be characterized as *natural monopolistic competition between differentiated pure public good producers with free entry.*

One might speculate that producing a relatively small number of such goods, each of which is consumed by a relatively large number of consumers, would be efficient. Doing so takes advantage of the lack of rivalry in consumption and spreads first copy costs more widely. This is not, however, what we see in practice. Instead, we see a proliferation of news, entertainment, literature, music, and videos, delivered through the web.

While each information good is a unique product, popular ones are quickly imitated. When markets are crowded, consumers are often able to choose among information goods that are close substitutes in their own estimation. At least in the absence of complicating network externalities, this implies that the demand for any given piece of content is self-limiting. If too many agents consume a given information good, it creates a market opportunity for a firm to enter and produce a slightly differentiated product that peels-off enough of the incumbent's user-base to cover costs.

We would also expect that preferences over goods are correlated between agents. Preferences of individuals are not random, and tend to rank commodities with similar characteristics similarly. For example, some people think that Beethoven's fifth symphony is one of the greatest orchestral pieces ever written, while others prefer the works of Mozart or Wagner. It is likely that a significant fraction of fifth symphony fans will also put Beethoven's third high in their ranking. We would expect that a much smaller share of Götterdämmerung fans would make a similar assessment.

In the end, the demand of individual users for information goods is bounded. Consumers have finite attention spans (or endowments of time), and simply do not have the capacity to consume an infinite amount of content. In addition, each time a user starts to consume a new information good, he incurs an initiation, or switching cost. This may be due to the effort involved in clicking a mouse, retrieving a book from a shelf, or simply refocusing his attention. Eventually, this cost becomes prohibitive, and as a result, it can only be optimal to consume at most a finite set of information goods.

Putting this together, we argue that the consumption side of an information economy has the following features:

- Taste diversity: We are all individuals. While it is likely that many of agents will enjoy similar content, it is unlikely that a large fraction of a population will agree that a given piece of content is perfect, and that no alternative could possibility be better.
- Correlated tastes: Preferences are not entirely random. Agents like content, and any commodity for that matter, because of the hedonic characteristics it bundles.
- Bounded attention spans: Even if content was free, agents have neither the interest nor the time to consume more than limited quantity. In addition, the costs of finding content and switching between offerings implies a practical limit to how many distinct pieces of content agents can consume.

In other words, the demand side can be characterized by *consumers with diverse, but correlated tastes, and bounded attention spans.*

This paper proposes a model of an information economy with a countably infinite number of consumers and producers. Each producer who chooses to enter the market provides a unique pure public good. On the other hand, the population of consumers is characterized by correlated taste diversity, and bounded attention spans.

We show that at any Pareto optimal (or approximately Pareto optimal) allocation, agents will consume a finite number of public goods, and that each of the goods that are produced will be consumed by a finite number of agents.

We explore two possible price systems in an attempt to decentralize these Pareto optimal allocations. The first assigns one (anonymous) price to each information good. These prices necessarily do not reflect the differing marginal willingness to pay for public goods across agents. Not surprisingly, anonymous prices fail to deliver the standard Welfare Theorems for the same reasons as in finite public goods economies. We then proceed more directly along the lines of Lindahl (1919) by allowing personalized prices. Despite assuming that firms are fully informed about agents' preferences, and are able to impose first degree price discrimination, the First Welfare Theorem continues to fail.

The inability of price systems, even ones with extreme informational requirements, to deliver Welfare Theorems suggests that this environment has inescapable complexities that prevent market forces from removing arbitrage opportunities. The information economy may be fundamentally entrepreneurial and creative in nature. Production, in particular, cannot be decentralized by any reasonable price system.

The paper proceeds as follows: Sections 2 discusses the connections between this paper and the literatures extending from the work of Tiebout and Buchanan. We describe our model in detail in Section 3, and characterize optimal allocations in Section 4. Section 5 confirms the failure of anonymous prices to decentralized efficient allocations. In Section 6 we define a notion called Entry-Free Project Equilibrium (EFPE) with nonanonymous prices. EFPE decentralizes consumer choice over goods, and the production choices of incumbent firms. It is also proof against entry from all potential firms, even when we allow them the power of first degree price discrimination. Unfortunately, we show that the First Welfare Theorem continues to fail. We discuss the relationship of our results to the innovation and entrepreneurship literature in Section 7, and the technical contribution of the paper in Section 8. Section 9 concludes.

2. Tiebout and Buchanan Literatures

Public goods result in market failure because of free-riding. Central planning also fails because agents' willingness to pay for public goods is private information. The great contribution of Tiebout (1956) and Buchanan (1965) was to offer a solution in which agents voluntarily sort themselves into coalitions, clubs, or locations. Each of these coalitions offers a bundle of public goods to agents who are willing to pay an admission price. Thus, agents vote with their feet, and thereby reveal their willingness to pay.

Both authors observe that many "public" goods are provided by cities, states, and clubs, and are neither purely rival, nor purely nonrival. The semi-rivalry of these goods implies that there is an optimal balance between the negative effects of too many agents trying to consume the same good at once, and the benefit of sharing the cost of public goods production more widely.

Buchanan proposed a framework of competitive clubs. Direct crowding or congestion effects that members impose on one another limit the optimal size of his clubs. For example, swimming pools, golf courses, and private schools, yield less benefit to each consumer as use approaches their capacities.

In contrast, Tiebout proposed a framework of jurisdictions tied to physical locations. Crowding might be direct, or a result of the relative scarcity of land close to an amenity such as park, or a fire station, whose consumption value decreases with distance.

Tying consumption to location also suggests that equilibrium should result in a partition of agents, each living in one, and only one, location providing a bundle of public goods. In contrast, agents might join several Buchanan-type clubs, each providing a different semi-rival good instead of a bundle.

A large literature studying semirival goods in different contexts has since developed. For example, crowding might be nonanonymous in that different types of agents may crowd each other differently. These crowding characteristics may even be endogenously chosen in response to market signals (see Conley and Wooders 2001). Congestion might depend on the number of visits each member makes to a club. The relative location and size of jurisdictions, or limitations on tax instruments available to jurisdictions, may also play a role in outcomes. See Sandler (2023) for a recent survey.

On the theory side in particular, there are many explorations of the nature of the core, competitive equilibrium allocations, and the associated price systems. See Allouch, Conley, and Wooders (2009), or Chan and van den Nouweland (2023) for recent treatments and discussions of this literature.

The current paper both follows and departs from the Buchanan and Tiebout frameworks. The key feature of these three approaches is summarized below:

- Multiple memberships – Buchanan, Conley
- Partition of agents in equilibrium – Tiebout
- Free-entry of producers – Tiebout, Conley
- General equilibrium markets – Tiebout, Conley
- Pareto optimality of small jurisdictions/memberships/subscriptions for each location/club/information good – Buchanan, Tiebout, Conley
    - Implied by semi-rivalry due to crowding and congestion externalities – Buchanan, Tiebout
    - Implied by semi-rivalry due to location and distance – Tiebout
    - Implied by limited attention spans and taste diversity for non-rival goods – Conley

We will delay the discussion of theoretical contribution of the current paper until after the model and results have been described.

3. The Model

Consider an economy with a countably infinite set of **Agents**,

$$i \in \mathcal{I} \equiv \mathbb{N}_+ \equiv \{1, \ldots\}.$$

We will use the convention that capital letters represent subsets. Thus, $i \in I \subset \mathcal{P}(\mathcal{I})$ means that agent i is in the coalition I which is an element of the power set of all agents.

The economy has one, infinitely divisible, **Private Good**,

$$x \in \mathbb{R}_+.$$

The initial allocation of the private good over agents is given by an **Endowment Map**,

$$\Omega: \mathcal{I} \Rightarrow [0, \overline{\Omega}]$$

where $\Omega(i)$ is the endowment of agent i. Note that we assume that this is bounded from above and below.

A **Private Good Allocation Map** is denoted,

$$X: \mathcal{I} \Rightarrow [x, \bar{x}]$$

where $X(i) = x_i$ is the private good allocation of agent i, and

$$X\_MAP$$

denotes the set of all possible private good allocation maps.

The economy also includes a countably infinite set of potential **Pure Public Goods**, which we will refer to as **Web Projects** or just, **Projects**, below,

$$w \in \mathcal{W} \equiv \mathbb{N}_+$$

where $w \in W \subset \mathcal{P}(\mathcal{W})$ means that project w is in a set, $W = \{\overline{w}, \ldots, w, \widetilde{w}, \ldots\}$, of web projects, which is an element of the power set of all web projects, and which may be finite or countably infinite. Note that we do not impose any Euclidean structure on the project/public good space.[3]

The cost of producing a project in terms of private good is given by a bounded **Cost Function**,

$$\mathrm{Cost}: \mathcal{W} \Rightarrow [0, \bar{c}]$$

where $\mathrm{Cost}(w) = c$ means that $c \leq \bar{c}$ is the private good cost of producing project $w \in \mathcal{W}$.

Each agent is assigned, or chooses, a set of **Subscriptions** to projects which collectively define a **Subscription Map** denoted,

$$\mathrm{SubMap}: \mathcal{I} \Rightarrow \mathcal{P}(\mathcal{W})$$

where $\mathrm{SubMap}(i) \equiv W_i \in \mathcal{P}(\mathcal{W})$ means that agent i subscribes to all web projects $w \in W_i$, and

3 One might pursue a hedonic approach along the lines of Lancaster (1966) and Rosen (1974). Here, the new commodities would be composed of different proportions of an underlying set of characteristics that are valued by consumers. Leaving aside the real world difficulties of understanding what these hedonic characteristics might be (more cowbell?), or if they would be of finite or countable in dimension, it is very unlikely that preferences would be convex or even monotonic in the underlying hedonic space. In turn, it is difficult to see how optimal commodity creation could not be decentralized by linear prices.

$$\text{SUB\_MAP}$$

denotes the set of all possible subscription maps.

Symmetrically, each project w that is produced is chosen for subscription by a set of **Users** which implies a **User Map** denoted,

$$\text{UserMap}: \mathcal{W} \Rightarrow \mathcal{P}(\mathcal{I})$$

where $\text{UserMap}(w) = I_w \in \mathcal{P}(\mathcal{I})$ means that project w is subscribed to by all agents $i \in I_w$, and

$$\text{USER\_MAP}$$

denotes the set of all possible user maps.

A subscription and user map, $\text{SubMap} \in \text{SUB\_MAP}$, and $\text{UserMap} \in \text{USER\_MAP}$, are said to be **Consistent** if $\forall\ i \in \mathcal{I}$, and $\forall\ w \in \mathcal{W}$,

$$i \in \text{UserMap}(w) \Leftrightarrow w \in \text{SubMap}(i).$$

The production of projects is paid for through a **Tax Plan** that collects enough private good from agents for each project to cover its costs,

$$\text{TaxPlan}: \mathcal{I} \times \mathcal{W} \Rightarrow \mathbb{R}_+$$

where $\text{TaxPlan}(i, w) = t_{i,w}$ means that agent i contributes $t_{i,w} \geq 0$ private good to the production of web project w and,

$$\text{TAX\_PLAN}$$

denotes the set of all possible tax plans. Note that a tax plan may require agents that contribute to projects to which they do not subscribe.

Given this, an **Allocation** is a triple,

$$\{X, \text{TaxPlan}, \text{SubMap}\} \in \text{X\_MAP} \times \text{TAX\_PLAN} \times \text{SUB\_MAP},$$

and is a **Feasible Allocation** given the endowment map Ω if,

1. $\forall\ i \in \mathcal{I}$, it holds that,

$$\Omega(i) - X(i) = \lim_{\bar{w} \to \infty} \sum_{w=1}^{\bar{w}} \text{TaxPlan}(i, w).$$

2. $\forall\ w \in \mathcal{W}$, such that $\text{UserMap}(w) \neq \emptyset$ it holds that,

$$\lim_{\bar{i} \to \infty} \sum_{i=1}^{\bar{i}} \text{TaxPlan}(i, w) = \text{Cost}(w).$$

where UserMap is consistent with SubMap.

■

The set of all possible feasible allocations for a given endowment map is denoted,

$$\text{FEASIBLE\_ALLOCATION}.$$

This definition says that:

(1) The difference between any agent's endowment and private good allocation equals the limit of the partial sums of his assigned tax contribution to web projects.

(2) For all web projects that are produced under a given subscription map (that is, have at least one subscriber), the cost of production equals the limit of the partial sums of the assigned tax contribution to the web project over agents. Note that this allows for the possibility that an agent might subscribe to an infinite number of projects, and that a project might have an infinite number of subscribers.

Each agent $i \in \mathcal{I}$ has a quasilinear **Utility Function** of the following form,

$$U_i(x_i, W_i) = x_i + V_i(W_i) - TC_i(W_i)$$

where,

x_i:	**Private Good** consumption of agent i.
$V_i(W_i)$:	**Transferable Utility** value to agent *i* of subscribing to all the projects in W_i.
$TC_i(W_i)$:	**Transaction Cost** to agent *i* of subscribing to all the projects in W_i.

The motivation for including a transaction cost for consuming a set of projects is that there are search, attention, and other fixed costs, associated with the act of calling up any given set of web pages, getting a set of books off the shelf, putting on a set of CDs, starting a game, or otherwise accessing content. This cost is independent of the time and effort required to consume and enjoy the content. Ultimately, it is this friction that prevents us from consuming an unlimited amount of content.[4] Note, however, that we do not impose monotonicity or convexity conditions on $V(W)$ or $TC(W)$.

We assume the following:

Assumption 1: $\forall\ i \in \mathcal{I}$, it holds that $V_i(\emptyset) = 0$, and $TC_i(\emptyset) = 0$.

Assumption 2: $\exists\ \bar{v} > 0$ such that $\forall\ i \in \mathcal{I}$ and $W_i \in \mathcal{P}(\mathcal{W})$, it holds that $V_i(W_i) \leq \bar{v}$.

Assumption 3: $\forall\ \bar{v} > 0$, $\exists\ \bar{n} \in \mathbb{N}_+$ such that $\forall\ i \in \mathcal{I}$, if $\| W_i \| > \bar{n}$, then it holds that $TC_i(W_i) > \bar{v}$, where $\| \bullet \|$ denotes the cardinality of a set.

Assumption 4: $\exists\ \bar{\varepsilon} > 0$ and $\bar{\delta} \in (0,1]$ such that,

$$\forall\ \text{UserMap} \in \text{USER\_MAP}$$

4 It might be interesting to generalize this by allowing all agents to receive both an increased flow of utility and an increased attention cost as they put more time or effort into consuming a given public project. This would allow us to consider the often neglected, but important, fact that one has only 24 hours a day to allocate not only to work and leisure, but also to consumption activities of all kinds. Consuming a leisurely dinner is different from wolfing one down. It may be more pleasurable to do the former, even with the same food in front of you, but it means that you must curtail how long you plays *Grand Theft Auto* later. We leave these considerations for future research.

$$\forall\ w \in \mathcal{W} \text{ such that UserMap}(w) = I_w \neq \emptyset,$$

$$\exists\ \overline{w} \in \mathcal{W} \text{ and } \bar{I} \subseteq I_w$$

such that

$$\| \bar{I} \| \geq \bar{\delta} \times \| I_w \| - 1,$$

and

$$\forall\ j \in \bar{I},$$

it holds that

$$j \notin \text{UserMap}(\overline{w})$$

$$V_j(W_j) - TC_j(W_j) < V_j(W_j \cup \overline{w} / w) - TC_j(W_j \cup \overline{w} / w) - \bar{\varepsilon}.$$

where $(W_j \cup \overline{w} / w)$ is the set of web projects W_j with project $\overline{w}$ added, and project w deleted.

Assumption 1 is just a normalization that says if agents do not subscribe to any projects, they receive no consumption benefits, and pay no attention/transactions costs.

Assumption 2 says that there is an upper limit on the utility that agents gets from any set of subscriptions. To allow otherwise would be to imagine that agents can achieve Nirvana while consuming a finite set of information goods, or perhaps approach it as they consume projects without bound. While it certainly is conceivable that agents might attain enlightenment by reading a book that contains the universal truth, or that listening to the Dark Side of the Moon enough times brings one close to a bliss point, the economic problem disappears in these cases. Note that Assumption 2 implies that $V_i(W_i) - TC_i(W_i)$ is bounded above.

Assumption 3 says that at some point, the attention cost of consuming one more web project exceeds any possible gain. We call this the "go to bed" constraint.

Assumption 4 is our correlated taste diversity condition. At a high-level, it says the following:

For every project $w \in \mathcal{W}$ *that is subscribed to by a non-empty coalition of agents* I_w

there exists

an alternative project $\overline{w} \in \mathcal{W}$, *and subcoalition* $\bar{I} \subseteq I_w$ *of agents*

such that

all agents in $\bar{I}$ *are at least* $\bar{\varepsilon} > 0$ *better-off if project* $\overline{w}$ *replaces project* w *in their set of project subscriptions.*

At a more detailed level, Assumption 4 also requires that:

- $\| \bar{I} \| \geq \bar{\delta} \times \| I_w \| - 1$, where, $\bar{\delta} \in (0, 1]$.
- $\bar{\varepsilon}$ and $\bar{\delta}$ are fixed parameters that hold for all allocations.

- No agent in subcoalition $\bar{I} \subseteq I_w$, is currently a subscriber to the alternative project $\overline{w}$.

Assumption 4 requires both a diversity of tastes, and the existence of commonly agreed upon substitutes. Diversity of tastes is implied by the existence of the subcoalition $\bar{I}$ that slightly prefers $\overline{w}$ to w in its set of subscriptions. The existence of commonly preferred alternative is implied by the requirement that all agents in a subcoalition of agents who were originally consuming project w agree that a specific alternative project $\overline{w}$ is slightly preferred (as opposed to requiring that all the agents in the subcoalition can individually find *some* project they slightly prefer).

Assumption 4 is meant to capture the idea that, while people are different, they tend to differentiate in similar ways. This assumption drives the key finding of Lemma 2, namely that at all Pareto optimal allocations, there is a finite upper bound on the number agents that subscribe to any given project.

Note that Assumption 4 also requires that subcoalition $\bar{I}$ must contain at least $\bar{\delta} \times \| I_w \| - 1$ agents. Since $\bar{\delta}$ can be very small, this will only bite, in general, if a very large number of agents are consuming a given project w. For example, if $\bar{\delta} = .0001$ and $\| I_w \| = 9{,}000$, then the size of subcoalition $\bar{I}$ would have to be at least $.9 - 1 = -.1$ agents, that is, zero agents.[5]

4. Optimal Allocations

A feasible allocation,

$$\{X, TaxPlan, SubMap\} \in FEASIBLE\_ALLOCATION$$

is **Pareto Optimal** if

$$\nexists\ \{\overline{X}, \overline{TaxPlan}, \overline{SubMap}\} \in FEASIBLE\_ALLOCATION$$

such that

$$\forall\ i \in \mathcal{I},$$
$$U_i(\overline{X}(i), \overline{W}_i) > U_i(X(i), W_i).$$

Note that we use the strict inequality in this definition because of the quasilinear structure of the utility functions. If a weak Pareto improvement exists, then it is possible to transfer small amounts of utility from agents who strongly prefer a weakly Pareto improving allocation, to agents who are indifferent, so that all strongly prefer the new allocation.

A feasible allocation,

$$\{X,\ TaxPlan,\ SubMap\} \in FEASIBLE\_ALLOCATION$$

is **ε-Pareto Optimal** if

5 Put another way, when $\| I_w \|$ is small, assumption 4 may allow $\| I_w \| = 0$. Note that in this case, however, it is already the case the number of agents consuming project w is small. Assumption 4 is only used to prove Lemma 2, which says that there is a bound on $\| I_w \|$, and so only needs to bind when number of agents consuming project w is large.

$$\nexists \{\overline{X}, \overline{TaxPlan}, \overline{SubMap}\} \in FEASIBLE\_ALLOCATION$$

such that

$$\forall\ i \in \mathcal{I},$$
$$U_i(\overline{X}(i), \overline{W}_i) > U_i(X(i), W_i) + \varepsilon.$$

The first Lemma says that there is a uniform upper bound on the number of projects that any agent will choose to consume at any Pareto optimal allocation.

Lemma 1. $\exists\ \bar{n} \in \mathbb{N}_+$ *such that for all Pareto optimal allocations,*

$$\{X, TaxPlan, SubMap\} \in FEASIBLE\_ALLOCATION,$$

and $\forall\ i \in \mathcal{I}$, *it holds that* $\| W_i \| < \bar{n}$, *where* $SubMap(i) = W_i$.

Proof: See appendix.

■

The second Lemma says that there is a uniform upper bound on the number of agents who will choose to subscribe to any given project at any Pareto optimal allocation.

Lemma 2. $\exists\ \bar{n} \in \mathbb{N}_+$ *such that for all Pareto optimal allocations,*

$$\{X, TaxPlan, SubMap\} \in FEASIBLE\_ALLOCATION,$$

and $\forall\ w \in \mathcal{W}$, *it holds that* $\| I_w \| < \bar{n}$, *where* $UserMap(w) = I_w$ *and* UserMap *is consistent with* SubMap.

Proof: See appendix.

■

Put together, Lemmas 1 and 2 say that all Pareto optimal allocations in an economy that satisfy Assumptions 1 through 4 will have the properties outlined in the introduction. In particular, at all Pareto optimal allocations, all agents will subscribe to a finite number of projects, and all projects that are produced will be subscribed to by a finite number of agents.

Our first Theorem shows that there will always exist a Pareto optimal allocation, at least in an ε sense.

Theorem 1. For all $\varepsilon > 0$, there exists an $\varepsilon-$Pareto optimal allocation.

Proof: *See appendix.*

■

The significance of this Theorem is to show that the attempt to prove Welfare Theorems for this economy is not vacuous. Pareto optimal allocations exist, at least in an ε sense. Discovering whether they can be decentralized by markets is therefore a worthwhile exercise.

5. Anonymous Prices

A **Price System** is a map from each agent and project to the nonnegative real numbers, plus the empty set,

$$Price: \mathcal{I} \times \mathcal{W} \Rightarrow \mathbb{R}_+ \cup \emptyset$$

where $\text{Price}(i,w) = p_{i,w}$ means that if agent i wishes to subscribe to project w, he must pay $p_{i,w}$ in private good to firm w, and

$$\text{PRICE}$$

denotes the set of all possible price systems.

Firms in this economy are each monopoly producers of a single potential public project. Given a price system, each firm w must decide whether to produce their project, w. More formally, a **Production Plan** for an economy is denoted,

$$\text{ProdPlan}: \mathcal{W} \Rightarrow \{0,1\}$$

where $\text{ProdPlan}(w) = 1$ means that firm w chooses to produce project w while $\text{ProdPlan}(w) = 0$ means that it chooses not to produce w, and

$$\text{PROD\_PLAN}$$

denotes the set of all possible production plans. Projects that are not produced, are also not priced,

$$\text{ProdPlan}(w) = 0 \Leftrightarrow \forall\ i \in \mathcal{I},\ \text{Price}(i,\ w) = \emptyset .$$

That is, agents are not offered the chance to subscribe to projects that are not produced in equilibrium.

The most natural approach to decentralization is to use anonymous prices. Otherwise, we would need to assume that firms can figure out each consumer's willingness to pay for projects, and are also able to impose them as prices on individuals. Although each firm is a monopoly producer, identification, and the prevention of resale, are strong assumptions it would be preferable to avoid.

A price system, $\text{Price} \in \text{PRICE}$, is said to be an **Anonymous Price System** if $\forall\ w \in \mathcal{W}$, and $\forall\ i,\ j \in \mathcal{I}$, holds that,

$$\text{Price}(i,w) = \text{Price}(j,w).$$

Unfortunately, anonymous decentralization is not generally possible, as the next Theorem demonstrates.[6]

Theorem 2. It is not possible to decentralize every Pareto optimal allocation with an anonymous price system.

Proof: Consider the following example. Suppose for each pair of adjacent agents, i and $i + 1$, where *i* is an odd number,

$$V_i(\mathbb{W}) = 11 \text{ and } V_{(i+1)}(\mathbb{W}) = 21 \text{ if } \mathbb{W} = \{w\} \text{ where } w = i$$

$$V_i(\mathbb{W}) = 0 \quad \text{and } V_{(i+1)}(\mathbb{W}) = 0 \quad \text{if } \mathbb{W} \neq \{w\} \text{ where } w = i$$

That is, each agent in these adjacent pairs, i and $i + 1$, enjoys positive utility only if he subscribes to the single web project $w = i$. Also suppose,

6 Note that we do not formally define an equilibrium notion here in the interest of space. The Theorem shows that regardless of how equilibrium is constructed, anonymous prices cannot provide the required market signals.

$$\forall\ i \in \mathcal{I},\ TC_i(W) = \| W \|$$

and

$$\forall\ w \in \mathcal{W},\ \text{Cost}(w) = 25.$$

Given this, the Pareto optimal allocation produces each of the odd numbered projects, $w \in \mathcal{W}$, and assigns subscriptions to agents $i = w$ and $i = w + 1$. Doing so generates a net transferable utility surplus of $(11 - 1) + (21 - 1) - 25 = 5$ from each adjacent pair of agents after production costs are paid, and transaction costs deducted. The associated tax system that completes the feasible allocation could be anything that divides the production cost of 25 between these pairs of agents, for example. Even numbered projects, on the other hand, would not be produced.

Unfortunately, there is no anonymous price system which can decentralize this allocation. Consider any odd numbered project $w \in \mathcal{W}$.

- If $\text{Price}(i,w) = \text{Price}(i+1,w) > 20$, then no agent chooses to subscribe to w, and so revenue equals zero.
- If $\text{Price}(i,w) = \text{Price}(i+1,w) \in (10,20]$, then agent $i+1$ alone chooses to subscribe to w, and so revenue equals $\text{Price}(i+1,w) \in (10,20]$,
- If $\text{Price}(i,w) = \text{Price}(i+1,w) \leq 10$, then both agents i and $i+1$ chose to subscribe to w, and so revenue equals $\text{Price}(i,w) + \text{Price}(i+1,w) \in [0,20]$.

Obviously, no agent would choose to subscribe to any even numbered project at any positive price. Thus, no anonymous price system that induces agents to choose the Pareto optimal set of subscriptions ($\text{Price}(i,w) = \text{Price}(i+1,w) \leq 10$) collects enough revenue to cover the project cost of 25, which is necessary to incentivize firm w to choose to produce the project. We conclude that is not possible to decentralize every Pareto optimal allocation with an anonymous price system.

■

Theorem 2 tells us that the Second Welfare Theorem fails for anonymous equilibrium. Perhaps this should not be surprising given that our projects are nonrival in consumption, and we would therefore expect that personalized Lindahlian prices should be required for decentralization.

A key insight of the Tiebout literature, however, is that decentralization in local public goods economies is possible with anonymous prices. Producers of jurisdictions compete with one another to provide bundles of public goods at take it or leave it prices. Agents reveal their willingness to pay by voting with their feet.

The information economy we outline in this paper also imagines producers competing with one another to provide public projects at take it or leave it anonymous prices. Further we consider a limiting case of assortative matching between an infinity of users, and infinity of projects, and show that neither agents nor producers have market power. Theorem 2 shows that despite the similarities between this information economy and local public goods or club economies, we do not get an analogous revelation result.

It is easy to use a slight variation of the example above to construct a failure of the First Welfare Theorem. Suppose,

$$\forall\ w \in \mathcal{W},\ \text{Cost}(w) = 15.$$

Then $\text{Price}(i,w) = \text{Price}(i+1,\ w) = 15$ results in agent $i+1$ choosing to subscribe to project $w = i$ and receiving a surplus of 5. Agent i, on the other hand, chooses not to subscribe to anything. If he did so at this price, he would receive -5 units of utility. Since the all Pareto optimal allocations require that *both* agents to subscribe to project $w = i$, we see that anonymous prices can decentralize Pareto dominated allocations. Thus, both the First and Second Welfare Theorems fail for anonymous prices.

6. Equilibrium and Efficiency

In our construction, projects which are not produced, are not priced:

$$\text{ProdPlan}(w) = 0 \Leftrightarrow \forall\ i \in \mathcal{I},\ \text{Price}(i,\ w) = \emptyset\,.$$

This means that our price system embeds incomplete markets, and so, perhaps, it should not be surprising that markets fail. Exactly how to complete markets, however is not immediate.

The economy defined here contemplates a countable infinity of projects that might, or might not, be produced in equilibrium. A complete price system, therefore, is tasked not only with allocating consumers to projects, but also decentralizing innovation in the sense that it optimally incentivizes the introduction of new commodities to the goods space. A complete price system would seem to require posting a marginal willingness to pay, for an infinite number of agents, for every conceivable variation of a web project.

It seems unlikely that anyone would bother to investigate the economic viability of such obviously undesirable potential projects as David Hasselhoff sings Mongolian throat music, on ice!, or a 37 volume work on the life and times of a hamster that Che Guevara owned as child. Some projects are not even worth considering from the standpoint of the economy as whole, much less deriving, and posting, a set of personalized prices for every agent in the economy.

A less exhaustive approach would be to imagine producers as entrepreneurs who conduct marketing studies to determine if there is enough willingness to pay in an economy to cover the costs of the potential project that they are capable of producing. In effect, each firm searches for a set of non-anonymous prices that would justify its entry into the market. These prices, however, do not become part of the equilibrium price system unless the firm actually enters.

This approach describes a type of “entry-free” equilibrium notion. Each firm holds constant the prices of incumbent firms, and the subscription choices of all agents. Given this, firms try to discover if there are prices that would induce enough agents to add their potential project to their existing set of subscriptions to cover the cost of production. Thus, incumbent firms are price-takers, but potential entrants are price-makers.

As price-takers, incumbent firms assume all agents assigned a positive price will subscribe to their project in equilibrium,

$$\text{Price}(i,w) > 0 \Leftrightarrow w \in \text{SubMap}(i).$$

Firms therefore decide to produce their project if the sum of these prices covers costs,

$$\text{ProdPlan}(w) = 1 \Leftrightarrow \lim_{\bar{i} \to \infty} \Sigma_{i=1}^{\bar{i}} \ \text{Price}(i,w) \geq \text{Cost}(w).$$

In general, firms may make profits, and so these would need to be returned to agents through a system of ownership shares. This would complicate the definition of equilibrium significantly. Since agents' utility functions are quasilinear, transfer of profits to agents would have no effect on either the equilibrium subscription choices, or the set of Pareto optimal allocations. We therefore simplify by constraining the equilibrium price system to sum to the costs of each of the projects that are produced in equilibrium in the definition below.

An **Entry-Free Project Equilibrium (EFPE)** with respect to an endowment map Ω consists of a triple,

$$\text{Price} \in \text{PRICE}$$

$$\text{ProdPlan} \in \text{PROD\_PLAN}$$

$$\{X, \text{TaxPlan}, \text{SubMap}\} \in \text{FEASIBLE\_ALLOCATION}$$

such that,

1. $\forall\ i \in \mathcal{I}$,

$$X(i) = \Omega(i) - \lim_{\bar{w} \to \infty} \Sigma_{w=1}^{\bar{w}} \text{Price}(i,w) = \Omega(i) - \lim_{\bar{w} \to \infty} \Sigma_{w=1}^{\bar{w}} \text{TaxPlan}(i,w)$$

2. $\forall\ i \in \mathcal{I}$, $\nexists\ \overline{\text{SubMap}} \in \text{SUB\_MAP}$ such that

$$U_i(\bar{x}_i, \overline{W}_i) > U_i(X(i), W_i)$$

where

$$\bar{x}_i = \Omega(i) - \lim_{\bar{w} \to \infty} \Sigma_{w=1}^{\bar{w}} \ \text{Price}(i,w), \ \text{and} \ \overline{W}_i = \overline{\text{SubMap}}(i)$$

3. $\forall\ w \in \mathcal{W}$,

$$\text{ProdPlan}(w) = 1 \Leftrightarrow \lim_{\bar{i} \to \infty} \Sigma_{i=1}^{\bar{i}} \ \text{Price}(i,w) = \text{Cost}(w)$$

$$\text{ProdPlan}(w) = 0 \Leftrightarrow \lim_{\bar{i} \to \infty} \Sigma_{i=1}^{\bar{i}} \ \text{Price}(i,w) < \text{Cost}(w)$$

4. $\forall\ \bar{w} \in \mathcal{W}$ such that $\text{ProdPlan}(\bar{w}) = 0$, $\nexists\ \overline{\text{Price}}(i,\bar{w})$ such that,

$$\lim_{\bar{i} \to \infty} \Sigma_{i=1}^{\bar{i}} \ \overline{\text{Price}}(i,\bar{w}) = \text{Cost}(w),$$

and $\forall\ i \in \mathcal{I}$ such that $\overline{\text{Price}}(i,\bar{w}) > 0$,

$$V_i(W_i \cup \bar{w}) - \text{TransCost}_i(W_i \cup \bar{w}) - \overline{\text{Price}}(i,\bar{w}) > V_i(W_i) - \text{TransCost}_i(W_i)$$

■

Condition 1 says that, for all agents, the equilibrium subscription plan and private good allocation are affordable. Note that this implies that

$$\lim_{\bar{w}\Rightarrow\infty} \sum_{w=1}^{\bar{w}} \text{Price}(i,w), \text{ and } \lim_{\bar{w}\Rightarrow\infty} \sum_{w=1}^{\bar{w}} \text{TaxPlan}(i,w),$$

must be finite even if the number of projects that an agent subscribes to, $\| W_i \|$, is not bounded.

Condition 2 says that, for all agents, the equilibrium subscription plan is optimal given prices. In particular, for each agent $i \in \mathcal{I}$, there does not exist an alternative subscription map, $\overline{\text{SubMap}}$, that leaves him better off under price system, Price, than the equilibrium subscription map, SubMap.

Condition 3 says that firms produce all projects for which the sum of prices over agents equals the cost of the project. Projects for which prices sum to less than cost are not produced. Note that incumbent firms that produce in equilibrium are price-takers.

Condition 4 says that firms enter the market if and only if they can find a set of prices for agents that would cause agents to add the firm's potential project to their subscription map, and which would cover the cost of production. Thus, the equilibrium is "entry-free" in the sense that even if all non-incumbent firms were price-makers capable of perfect first-degree price discrimination, profitable entry by any potential firm is infeasible. We discuss this modeling choice in more detail at the end of this Section.

The next Theorem says that even with non-anonymous prices and first degree price discrimination, it is still possible to support Pareto dominated allocations as equilibria.

Theorem 3. EFPE allocations may or may not be Pareto optimal.

Proof: *See appendix.*

■

The equilibrium notion defined above extends the standard approach used in general equilibrium literature. Specifically, firms are modeled as non-strategic agents that maximize profits unilaterally with respect to existing market conditions.

We see that despite our equilibrium concept incorporating first-degree price discrimination, an infinite set of nonanonymous prices for an infinite set of goods, and price-making potential entrants for every possible new project, markets fail profoundly.

As we note in Section 3, we do not assume that preferences satisfy convexity or monotonicity. It is unlikely, for example, that agents would prefer to read the first half *Alice in Wonderland*, and the second half of *Macbeth*, to reading the entirety of either work. Similarly, viewers may enjoy each successive episode or season of a show more than the last as they become invested in well-written characters and story-lines. If a show continues after its logical end with contrived and derivative plots, however, viewers are likely to enjoy each successive episode less than the last. In short, anything is possible, and even reasonable, in this regard.

More generally, projects may have arbitrary complementarities and substitutabilities. For example, a movie about a new superhero might fail to be profitable on its own. If several creators were to jointly make movies that build an interesting universe and set intertwining storylines, however, then each of the movies in this complementary set might very well find profitable markets.

One might wonder if Welfare Theorems could be recovered if producers were allowed to join cartels and collude to offer each consumer a personalized bundle of projects, at a personalized price. In the absence of entry, a First Welfare Theorem, and a constrained Second Welfare Theorem could probably be proved. Unfortunately, it is easy to construct examples in which colluding entrants could replicate a set of Pareto optimal bundles produced by colluding incumbents, and offer them at slightly lower personalized prices. Equilibrium may not exist in such cases. It is also possible to construct examples where entrants create cycles over non-Pareto optimal allocations.

Summing up, allowing firms to collude and take advantage of project complementarities would not lead to Welfare or Existence Theorems unless we also fixed the commodity space, and therefore did not allow entry. This does not reflect observed market conditions for information goods. It also requires giving firms even more unrealistic knowledge of consumer willingness to pay, and the ability to sort through all feasible bundles of projects. Consequently, we do not explore equilibrium extensions in this direction.

7. Innovation and Entrepreneurship

Arrow-Debreu-McKenzie economies, and their extensions to public goods and local public goods, generally embed the idea of a fixed commodity space. The function of prices is to govern efficient production and consumption of these commodities.

Millions of websites, videos, blog posts, songs, movies, books, and so on, appear every day, and there is no pre-existing description of such commodities for creators to reference. Exactly what kind of market signals content creators are responding to is not clear. Certainly, it is not a standard set of Walrasian prices. More likely, it relates to some estimate by creators of the demand for the content or product that they have in mind based on current and projected consumer behavior.

Despite the proliferation of the internet and related research in fields such as computer science, surprisingly few theoretical papers have examined the creation knowledge goods. Crémer *et al.* (2000), Besen *et al.* (2001), and Laffont *et al.* (2001) study issues of pricing and bargaining between internet service providers and internet backbone providers, and Jackson and Rogers (2007) explore a simple model of network formation that captures observed regularities in the network of links between websites. None of these studies, however, focus on the actual provision of content by website operators.

Product innovation more generally is far from a new topic in economics. The Schumpeterian view emphasizes the importance of entrepreneurial activity (Schumpeter 1947). More recent empirical work demonstrates the importance of entrepreneurship in growing economies, see McMillan and Woodruff (2002), for example. Baumol (1968) argues that entrepreneurs should not be modeled as maximizers of some sort of objective. His belief is that inspiration is more important for the creation of new ideas or products than calculation.

It is clear that any reasonable model of an information economy for content and other creative works must treat the commodity space as endogenous. Armstrong (2006), for example, takes a step in this direction with his exploration of endogenous product differentiation of platforms, but does not extend it to the full product space.[7] Harper *et al.* (2005), in contrast, consider the creation of crowdsourced content, and explores the incentives for users to contribute to an online movie review website. Their approach, however, is purely decision-theoretic and does not consider market or strategic interactions.

Other theoretical work includes Nelson and Winter (1974, 1977, and 1978), who use an off-equilibrium dynamic selection approach rather than neoclassical equilibrium analysis, Lucas (1978), Kihlstrom and Laffont (1979), and Calvo and Wellisz (1980), who explore the self-selection of entrepreneurs based on skill and risk aversion, and Aron and Lazear (1990) who analyze the decision to enter into the production of a new good. In general, this literature tends to abstract from pricing and other market signals, and does not consider what types of new goods are created, or why.

It is worth noting that while the recent theoretical work on optimal and equilibrium innovation is limited, the applied and policy literatures are more active. See Jullien and Sand-Zantman (2021), Jullien and Pavan (2018), and Mazzucato (2021) for recent discussions. Where prices are considered, however, it is more often from the standpoint of using artificial intelligence and other new tools to maximize profits and choose discriminatory prices for existing information goods. Thus, the focus is not on equilibrium outcomes. See Lerner (2020), Shiozawa (2020) and Steel *et a*l. (2021), and references therein.

8. Countably Infinite Public Goods Economies

Aumann (1964) was the first to present a model of a private goods economy with a continuum of agents. He argues that it is only in such an environment that the competitive assumption that agents are price takers is fully justified. The attraction of his approach is that it provides a very clean mathematical structure to gain insights about large finite economies. For example, he was able to show that Debreu and Scarf's (1963) result that the core converges to the competitive allocations as the economy gets large holds exactly in the sense that the core and equilibrium allocations are equivalent in a continuum economy.

Expanding on the Lindahl approach for pricing public goods, Mas-Colell (1980) defines a valuation equilibrium, and a cost-share equilibrium, for partial equilibrium settings with a single private good and a finite set of public projects. This was generalized by De Simone and Graziano (2004)

7 Although the product development process is not well understood, ours is certainly not the first general equilibrium model with an infinite number of commodities. Classic models of savings and consumption allow agents to choose infinite streams of future consumption levels. For example Mas-Colell (1975) and Jones (1983) model private goods as a vector of characteristics and consumers' consumption bundles as measures over characteristic space. These models assume an infinite number of agents and no production. We will not attempt to survey this very large literature here, but see Podczecka and Yannelis (2007) for a recent treatment with references and Aliprantis *et al.* (1990) for an older, but more comprehensive discussion of the literature. As far as we are aware, none of these models include public goods.

who include production and a Riesz space of private goods. Allouch, Conley, and Wooders (2009) further extend the continuum model to a Tiebout-like local public goods economy.

Aumann's pathbreaking work provides many useful insights precisely because the continuum economy he considered was clearly an economically meaningful limit case of a large finite economy. Without this connection, his results might simply have been artifacts of his modeling choices, and so would give misleading, or even incorrect, intuitions about large finite economies.

Unfortunately, the connection between the finite and continuum cases is much less clear for pure public goods economies. See Muench (1972), for the canonical treatment, and Berliant and Rothstein (2000) for a more recent treatment that discusses some issues outlined below. The central problem with the continuum approach to public goods economies is that private goods and public goods are measured in ways that make them fundamentally incomparable.

To see this, consider an economy with a fixed, finite set of public and private goods, and a continuum of consumers. Suppose that the average contribution of agents to public goods production is strictly positive. Since there are infinitely many consumers, there would have to be an infinite number of dollars to spend of public goods. How would one compare two such allocations? Is it meaningful to say that one infinity of public goods is preferred to another? Would we be better or worse off if we halved our contributions given that we would still be funding all public goods at infinite levels?

On the other hand, suppose that the average agents' contribution is zero. Would their Lindahl taxes be infinitesimal in this case? How would agents express demand for public goods when faced with infinitesimal Lindahl prices? In any event, how would we distinguish between different allocations in which public good levels are unmeasurable, or almost zero? For such allocations, the ratio of public to private goods consumption for each consumer could be bounded or undefined. Differentiating these two cases is important, but is not possible in a standard continuum economy.

We conclude that the direct extension of the Aumann approach to public goods creates significant mathematical and economic difficulties that make it hard to understand it as a limit of a large finite economy.

The main theoretical contribution of this paper is to develop an economy with a countable infinity of agents, goods, and producers, that solves the measure incompatibility problem described above. Agents make finite contributions, to the finite number of public goods they consume, while each public good produced is chosen by a finite set of consumers, at any optimal allocation.

At no point is it necessary to contemplate infinitesimals or infinities, something we argue is beyond the capabilities of consumers. More crucially, the model reflects a reasonable limiting case of a large public goods economy, and generates outcomes that match many features of the information economy in which we actually live. We are unaware of other countably infinite general equilibrium models with or without, public goods. It is countability, however, that makes it possible to define feasibly, the consumer’s problem, and to obtain the result presented in this paper.

9. Conclusions

This paper argues that it is unlikely that public goods consumption would grow without bound as an economy gets large. At some point, agents cease to be able to even to contemplate the vastness of such consumption bundles. Certainly, agents would not be able to comprehend infinities of information goods. However, if both the number and quantity of public goods is finite, the average private good contribution to project production (Lindahl taxes, for example) must converge to zero. This clearly is not happening in any real world economy we see, no matter how large.

We propose an alternative: as the economy gets large, the number of projects (which we think of as internet or information goods) also gets large. Thus, the commodity set is not fixed as it is in traditional Arrow-Debreu-McKenzie/Samuelson frameworks. We show under fairly mild conditions that the Pareto optimal allocations of this economy involve an infinite number of public projects being produced, each of which is consumed by a finite number of agents. In addition, each agent chooses to consume only a finite and bounded number of projects.

What drives these results are two basic assumptions: (1) agents have limited attention spans, and it costs agents a certain amount of effort to begin to consume each public project and (2) there is a diversity of tastes in the population, but also a degree of taste correlation between agents. There are likely to be close substitutes for any project that are at least slightly preferred, all else equal, by a fraction of any set of consumers. In effect, limited attention spans and diversity of tastes, turn a pure public goods economy into a club good economy, even though there is no crowding.

Although the optimal allocations are exactly what we believe we see in the real world, things get tricky when the question of how to support these allocations as equilibrium outcomes is considered. It is immediately clear that efficient decentralization is impossible using anonymous prices.

We offer an alternative, non-anonymous, equilibrium concept called Entry-Free Project Equilibrium. EFPE has extremely high information requirements and assumes that firms are able to engage in first-degree price discrimination. Despite this, Welfare Theorems fail. Even when the equilibrium is entry-free, it is still possible to support Pareto dominated allocations. Pareto improvements sometimes require coordinated, multilateral, entry, and significant shifts in demand behavior. Prices that correctly incentivize only unilateral entry of any potential firm turn out to be insufficient.

While this might be seen to be a negative result, we think it is better viewed as a positive conclusion. Information goods are infinitely variable, and have the kind of discrete, non-metrizable structure we describe in the paper. There is no sense in which a Taylor Swift song can be compared to a Bach cantata. At least we know of no hedonic or other quantitative metric that could do so. They are simply different.

The market, therefore, has a hard time signaling that a certain new public project should be produced. Entrepreneurs take educated guesses about what might succeed, but there are no arbitrage opportunities signaled by an equilibrium price system.

Thus, we argue that we generally should not expect to see first best outcomes in large information economies. It is possible to get rich (that is, make economic profits) if you happen to stumble on a public project that you can produce cheaply, and which happens to find a ready market. It should not be at all surprising that no one beat you to it. There may indeed be five dollar bills lying on the ground in the new information economy.

10. Mathematical Appendix

Lemma 1. $\exists\, \bar{n} \in \mathbb{N}_+$ *such that for all Pareto optimal allocations,*

$$\{X, \mathrm{TaxPlan}, \mathrm{SubMap}\} \in \mathrm{FEASIBLE\_ALLOCATION},$$

and $\forall\ i \in \mathcal{I}$, *it holds that* $\| W_i \| < \bar{n}$, *where* $\mathrm{SubMap}(i) = W_i$.

Proof: Suppose instead that no such upper-bound exists. Then for any $\bar{n} \in \mathbb{N}_+$, there must exist some agent $j \in \mathcal{I}$ such that,

$$\| \mathrm{SubMap}(j) \| = \| W_j \| > \bar{n}.$$

But by Assumption 2,

$$\exists\ \bar{v} > 0 \text{ such that } \forall\ i \in \mathcal{I} \text{ and } W_i \in \mathcal{P}(\mathcal{W}), \text{ it holds that } V_i(W_i) \le \bar{v},$$

and by Assumption 3,

$$\forall\ \bar{v} > 0,\ \exists\, \bar{n} \in \mathbb{N}_+ \text{ such that } \forall\ i \in \mathcal{I}, \text{ if } \| W_i \| > \bar{n}, \text{ then } TC_i(W_i) > \bar{v}.$$

We claim that this contradicts the hypothesis that $\{X, \mathrm{TaxPlan}, \mathrm{SubMap}\}$ is Pareto optimal. Consider the following alternative allocation which we claim is Pareto dominant,

$$\{\overline{X}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}}\} \in \mathrm{X\_MAP} \times \mathrm{TAX\_PLAN} \times \mathrm{SUB\_MAP},$$

where

$\overline{\mathrm{UserMap}}$ is compatible with $\overline{\mathrm{SubMap}}$, and

(1) $\forall\ i \in \mathcal{I}, i \neq j$

$$\overline{\mathrm{SubMap}}(i) = \mathrm{SubMap}(i)$$

and for agent $j \in \mathcal{I}$,

$$\overline{\mathrm{SubMap}}(j) = \emptyset.$$

(2) $\forall\ i \in \mathcal{I}$, and $\forall\ w \in \mathcal{W}$,

(a) if $(i)\ \overline{\mathrm{UserMap}}(w) \neq \emptyset$, then

$$\overline{\mathrm{TaxPlan}}(i, w) = \mathrm{TaxPlan}(i, w)$$

(b) if $\overline{\mathrm{UserMap}}(w) = \emptyset$, then

$$\overline{\mathrm{TaxPlan}}(i, w) = 0$$

(3) $\forall\ i \in \mathcal{I}$

$$\overline{X}(i) = \Omega(i) - \lim_{\bar{w} \to \infty} \textstyle\sum_{w=1}^{\bar{w}} \overline{\mathrm{TaxPlan}}(i, w).$$

First we show that $\{\overline{X}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}}\}$ is feasible. Note that all projects that are produced under SubMap are also produced under $\overline{\mathrm{SubMap}}$, except projects for which UserMap contains only agent j, (that is, $\mathrm{UserMap}(w) \equiv I_w = \{j\}$). For these non-produced projects with no subscribers under $\overline{\mathrm{SubMap}}$,

$$\overline{\mathrm{TaxPlan}}(j, w) = 0,$$

while for all other projects,

$$\overline{\mathrm{TaxPlan}}(i, w) = \mathrm{TaxPlan}(i, w).$$

Thus, $\forall\ w \in \mathcal{W}$, such that $\overline{\mathrm{UserMap}}(w) \neq \emptyset$ it holds that,

$$\lim_{\bar{i} \Rightarrow \infty} \sum_{i=1}^{\bar{i}} \overline{\mathrm{TaxPlan}}(i, w) = \mathrm{Cost}(w).$$

Part (3), above, requires that $\forall\ i \in \mathcal{I}$,

$$\bar{X}(i) = \Omega(i) - \lim_{\bar{w} \Rightarrow \infty} \sum_{w=1}^{\bar{w}} \overline{\mathrm{TaxPlan}}(i, w).$$

Thus, both conditions of the feasibility are satisfied, and so,

$$\{\bar{X}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}}\} \in \mathrm{FEASIBLE\_ALLOCATION}.$$

Next, we show that $\{\bar{X}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}}\}$ Pareto dominates $\{X, \mathrm{TaxPlan}, \mathrm{SubMap}\}$. Note that $\forall\ i \in \mathcal{I}$, and $w \in \mathcal{W}$,

$$\mathrm{TaxPlan}(i, w) \geq \overline{\mathrm{TaxPlan}}(i, w),$$

since taxes at both allocations are identical except for those projects that happen not to be produced under $\overline{\mathrm{SubMap}}$, and for which all taxes drop to 0. This implies for $\forall\ i \in \mathcal{I}$,

$$\Omega(i) - \lim_{\bar{w} \to \infty} \sum_{w=1}^{\bar{w}} \overline{\mathrm{TaxPlan}}(i, w) \equiv \bar{X}(i) \geq X(i).$$

Since $\forall\ i \in \mathcal{I}, i \neq j$,

$$\mathrm{SubMap}(i, w) = \overline{\mathrm{SubMap}}(i, w),$$

all agents besides agent *j* consume the same level of private good, and subscribe to the same set of projects, and so are indifferent between the two feasible allocations.

For agent *j*, by Assumptions 2 and 3,

$$TC_j(W_j) > \bar{v} \geq V_j(W_j) \Rightarrow V_j(W_j). - TC_j(W_j) < 0,$$

and since $\overline{\mathrm{SubMap}}(j) = \emptyset$,

$$V_j(\overline{W}_j) - TC_j(\overline{W}_j) = V_j(\emptyset). - TC_j(\emptyset) = 0.$$

Agent *j*, consumes at least the same level of private good in both feasible allocations, but receives more transferable utility from $\overline{\mathrm{SubMap}}(j, w)$ (which gives zero instead of negative utility).

We conclude that $\{\bar{X}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}}\}$ weakly Pareto dominates $\{X, \mathrm{TaxPlan}, \mathrm{SubMap}\}$. Since utility is quasilinear, it is possible to reallocate private good away from agent j, and to all other agents, such that all are strictly better off, which completes the proof.

■

Lemma 2. $\exists\ \bar{n} \in \mathbb{N}_+$ *such that for all Pareto optimal allocations,*

$$\{ \mathrm{X},\ \mathrm{TaxPlan},\ \mathrm{SubMap} \} \in \mathrm{FEASIBLE\_ALLOCATION},$$

and $\forall\ \mathrm{w} \in \mathcal{W}$, *it holds that* $\| \mathrm{I}_{\mathrm{w}} \| < \bar{\mathrm{n}}$, *where* $\mathrm{UserMap}(\mathrm{w}) = \mathrm{I}_{\mathrm{w}}$ *and* UserMap *is consistent with* SubMap.

Proof: Suppose instead no such upper-bound exists. Then for any $\bar{\mathrm{n}} \in \mathbb{N}_{+}$, $\exists\ \tilde{\mathrm{w}} \in \mathcal{W}$ such that,

$$\| \mathrm{I}_{\tilde{\mathrm{w}}} \| > \bar{\mathrm{n}}.$$

But by Assumption 4, $\exists\ \bar{\varepsilon} > 0,\ \bar{\delta} \in (0, 1]$, a subcoalition $\bar{\mathrm{I}} \subseteq \mathrm{I}_{\tilde{\mathrm{w}}} \equiv \mathrm{UserMap}(\tilde{\mathrm{w}})$, and a project $\bar{\mathrm{w}} \in \mathcal{W}$ such that,

$$\| \bar{\mathrm{I}} \| \geq \bar{\delta} \times \| \mathrm{I}_{\tilde{\mathrm{w}}} \| - 1,$$

and

$$\forall\ \mathrm{j} \in \bar{\mathrm{I}},$$

it holds that

$$\mathrm{j} \notin \mathrm{UserMap}(\bar{\mathrm{w}})$$

$$\mathrm{V}_{\mathrm{j}}(\mathbb{W}_{\mathrm{j}}) - \mathrm{TC}_{\mathrm{j}}(\mathbb{W}_{\mathrm{j}}) < \mathrm{V}_{\mathrm{j}}(\mathbb{W}_{\mathrm{j}} \cup \bar{\mathrm{w}} \,/\, \tilde{\mathrm{w}}) - \mathrm{TC}_{\mathrm{j}}(\mathbb{W}_{\mathrm{j}} \cup \bar{\mathrm{w}} \,/\, \tilde{\mathrm{w}}) - \bar{\varepsilon}.$$

Consider the following alternative allocation which we claim Pareto dominates $\{ \mathrm{X}, \mathrm{TaxPlan}, \mathrm{SubMap} \}$,

$$\{ \bar{\mathrm{X}}, \overline{\mathrm{TaxPlan}}, \overline{\mathrm{SubMap}} \} \in \mathrm{X\_MAP} \times \mathrm{TAX\_PLAN} \times \mathrm{SUB\_MAP},$$

where

(1) $\forall\ \mathrm{i} \notin \bar{\mathrm{I}}$,

$$\overline{\mathrm{SubMap}}(\mathrm{i}) = \mathrm{SubMap}(\mathrm{i})$$

and $\forall\ \mathrm{j} \in \bar{\mathrm{I}}$,

$$\overline{\mathrm{SubMap}}(\mathrm{j}) = \{ \mathbb{W}_{\mathrm{j}} \cup \bar{\mathrm{w}} \,/\, \tilde{\mathrm{w}} \}$$

(2) $\forall\ \mathrm{i} \in \mathcal{I}$, and $\forall\ \mathrm{w} \in \mathcal{W}, \mathrm{w} \neq \bar{\mathrm{w}}$

$$\overline{\mathrm{TaxPlan}}(\mathrm{i}, \mathrm{w}) = \mathrm{TaxPlan}(\mathrm{i}, \mathrm{w})$$

$\forall\ \mathrm{i} \notin \bar{\mathrm{I}}$,

$$\overline{\mathrm{TaxPlan}}(\mathrm{i}, \bar{\mathrm{w}}) = 0,$$

and $\forall\ \mathrm{j} \in \bar{\mathrm{I}}$,

$$\overline{\mathrm{TaxPlan}}(\mathrm{j}, \bar{\mathrm{w}}) = \mathrm{Cost}(\bar{\mathrm{w}}) \div \| \bar{\mathrm{I}} \|,$$

(3) $\forall\ \mathrm{i} \in \mathcal{I}$,

$$\bar{\mathrm{X}}(\mathrm{i}) = \Omega(\mathrm{i}) - \lim_{\bar{\mathrm{w}} \to \infty} \sum_{\mathrm{w}=1}^{\bar{\mathrm{w}}} \overline{\mathrm{TaxPlan}}(\mathrm{i}, \mathrm{w}).$$

Consider agents $\mathrm{i} \notin \bar{\mathrm{I}}$. Part (1), above, implies that for all such agents, the subscription map does not change. Part (2) says that $\forall\ \mathrm{i} \notin \bar{\mathrm{I}}$ and $\forall\, \mathrm{w} \in \mathcal{W}$, either $\overline{\mathrm{TaxPlan}}(\mathrm{i}, \mathrm{w}) = \mathrm{TaxPlan}(\mathrm{i}, \mathrm{w})$, or the tax contribution goes to zero, $\overline{\mathrm{TaxPlan}}(\mathrm{i}, \bar{\mathrm{w}}) = 0$. In all cases,

$$\mathrm{TaxPlan}(\mathrm{i}, \mathrm{w}) \geq \overline{\mathrm{TaxPlan}}(\mathrm{i}, \mathrm{w}),$$

Since Part (3) states that,

$$\bar{\mathrm{X}}(\mathrm{i}) = \Omega(\mathrm{i}) - \lim_{\bar{\mathrm{w}} \to \infty} \sum_{\mathrm{w}=1}^{\bar{\mathrm{w}}} \overline{\mathrm{TaxPlan}}(\mathrm{i}, \mathrm{w}),$$

we conclude that $\forall\ i \notin \bar{I}$,

$$U_i(\overline{X}(i), \overline{SubMap}(i)) \geq U_i(X(i), SubMap(i)).$$

Next consider agents $j \in \bar{I}$. Part (1) states that their subscription plans have changed only by the deletion of project $\tilde{w}$, and the addition of project $\bar{w}$. By Assumption 4,

$$V_j(W_j) - TC_j(W_j) < V_j(W_j \cup \bar{w} / \tilde{w}) - TC_j(W_j \cup \bar{w} / \tilde{w}) - \bar{\varepsilon} = V_j(\overline{W}_j) - TC_j(\overline{W}_j) - \bar{\varepsilon}.$$

Part (2) says that $\forall\ j \in \bar{I}$, and $\forall\ w \in \mathcal{W},\ w \neq \bar{w}$, $\overline{TaxPlan}(j,w) = TaxPlan(j,w)$. However, this set of agents also pay an additional tax for project $\bar{w}$ of

$$\overline{TaxPlan}(j, \bar{w}) = Cost(\bar{w}) \div \| \bar{I} \|.$$

Thus, both the taxes and the utility value of the subscriptions for agents $j \in \bar{I}$ have increased. In net, however, agents $j \in \bar{I}$ are better off if,

$$(V_j(\overline{W}_j) - TC_j(\overline{W}_j)) - (V_j(W_j)) - TC_j(W_j)) > \overline{TaxPlan}(j, \bar{w}).$$

In turn, this holds if,

$$\bar{\varepsilon} > Cost(\bar{w}) \div \| \bar{I} \|.$$

Recall, however, that $I_{\tilde{w}}$ can be chosen to contain an arbitrarily large number of users, and since,

$$\| \bar{I} \| \geq \bar{\delta} \times \| I_{\tilde{w}} \| - 1,$$

and $Cost(\bar{w}) \leq \bar{c}$, is bounded, $\| \bar{I} \|$ can also be made large enough so that

$$\bar{\varepsilon} > Cost(\bar{w}) \div \| \bar{I} \|.$$

Together, this implies that $\forall\ i \notin \bar{I}$, agent i is exactly as well off, while $\forall\ j \in \bar{I}$, agent j is strictly better off at $\{\overline{X}, \overline{TaxPlan}, \overline{SubMap}\}$. Since utility is quasilinear, it is possible to reallocate private good away from agent j, and to all other agents, such that all are strictly better off, which completes the argument.

It only remains to show,

$$\{\overline{X}, \overline{TaxPlan}, \overline{SubMap}\} \in FEASIBLE\_ALLOCATION.$$

To see this, note that $\forall\ i \in \mathcal{I}$, and $\forall\ w \in \mathcal{W},\ w \neq \bar{w}$, $\overline{TaxPlan}(i,w) = TaxPlan(i,w)$. Thus, for all projects besides $\bar{w}$,

$$\lim_{\bar{i} \Rightarrow \infty} \sum_{i=1}^{\bar{i}} TaxPlan(i,w) = \lim_{\bar{i} \to \infty} \sum_{i=1}^{\bar{i}} \overline{TaxPlan}(i,w) = Cost(w),$$

For project $\bar{w}$,

$$\forall\ j \in \bar{I}, \overline{TaxPlan}(j, \bar{w}) = Cost(\bar{w}) \div \| \bar{I} \|,$$

and

$$\forall\ i \notin \bar{I},\ \overline{TaxPlan}(i, \bar{w}) = 0.$$

Summing over all agents $i \in \mathcal{I}$ shows,

$$\lim_{\bar{i} \Rightarrow \infty} \sum_{i=1}^{\bar{i}} \overline{\text{TaxPlan}}(i, \overline{w}) = (\text{Cost}(\overline{w}) \div \| \bar{I} \|) \times \| \bar{I} \| = \text{Cost}(w).$$

Thus, the $\overline{\text{TaxPlan}}$ exactly pays for all public projects.

Part (3) requires $\forall\ i \in \mathcal{I}$,

$$\bar{X}(i) = \Omega(i) - \lim_{\bar{w} \to \infty} \sum_{w=1}^{\bar{w}} \overline{\text{TaxPlan}}(i, w),$$

which balances the budget, and complete the proof that $\{\bar{X}, \overline{\text{TaxPlan}}, \overline{\text{SubMap}}\}$ is feasible.

■

The **Mean Utility** of a feasible allocation is denoted,

$$\text{MeanUtility} : \text{FEASIBLE\_ALLOCATION} \Rightarrow \mathbb{R}_+$$

and defined as,

$$\text{MeanUtility}(X, \text{TaxPlan}, \text{SubMap}) \equiv \lim_{\bar{i} \to \infty} \sum_{i=1}^{\bar{i}} \frac{\left(X(i) + V_i(W_i) - TC_i(W_i)\right)}{\bar{i}}.$$

Theorem 1. For all $\varepsilon > 0$, there exists an ε – Pareto optimal allocation.

Proof: Define the set of mean utility values for set of all feasible plans as follows:

$$\{U\} \equiv \{U \in \mathbb{R}_+ \mid \exists \{X, \text{TaxPlan}, \text{SubMap}\} \in \text{FEASIBLE\_ALLOCATION}, \text{ and } U = \text{MeanUtility}(X, \text{TaxPlan}, \text{SubMap})\}.$$

By Assumption 2, the utility value of any set of subscriptions assigned by SubMap to any agent has an upper bound. Since endowments are also bounded above, MeanUtility of any feasible allocation is must also be bounded above. This implies that $\{U\}$ is a bounded above subset of $\mathbb{R}_+$, and it therefore must have a supremum, U^*. In turn, there must exist a sequence of feasible plans,

$$\{X^s, \text{TaxPlan}^s, \text{SubMaps}\} \subseteq \text{FEASIBLE\_ALLOCATION}$$

such that for any $\varepsilon > 0$, $\exists\ \bar{s} \in \mathbb{N}_+$ such that $\forall\ s > \bar{s}$,

$$\varepsilon > U^* - \text{MeanUtility}(X^s, \text{TaxPlan}^s, \text{SubMap}^s),$$

Now suppose for any $\varepsilon > 0$, that their existed a feasible allocation

$$\{\bar{X}, \overline{\text{TaxPlan}}, \overline{\text{SubMap}}\} \in \text{FEASIBLE\_ALLOCATION}$$

such that $\forall\ s > \bar{s}$, $\forall\ i \in \mathcal{I}$

$$U_i(\bar{X}(i), \overline{\text{SubMap}}(i)) > U_i(\bar{X}^s(i), \overline{\text{SubMap}}^s(i)) + \varepsilon.$$

That is, $\forall\ s > s$, $\{X^s, \text{TaxPlan}^s, \text{SubMap}^s\}$ ε – Pareto dominates $\{\bar{X}, \overline{\text{TaxPlan}}, \overline{\text{SubMap}}\}$. This implies,

$$\text{MeanUtility}(\bar{X}, \overline{\text{TaxPlan}}, \overline{\text{SubMap}}) > \text{MeanUtility}(X^s, \text{TaxPlan}^s, \text{SubMap}^s) + \varepsilon.$$

But,

$$U^* \geq \text{MeanUtility}(\overline{X}, \overline{\text{TaxPlan}}, \overline{\text{SubMap}})$$

which implies,

$$U^* > \text{MeanUtility}(X^s, \text{TaxPlan}^s, \text{SubMap}^s) + \varepsilon.$$

and so,

$$U^* - \text{MeanUtility}(\text{Xs}, \text{TaxPlans}, \text{SubMaps}) > \varepsilon,$$

contradicting the construction of the supremum, U^*.

■

Theorem 3. EFPE allocations may or may not be Pareto optimal.

Proof: Consider the following example. For each $n \in \mathbb{N}_+$, construct a coalition, I^n, consisting of ten adjacent agents as follows,

$$I^n \equiv \{10 \times (n-1)+1, \ldots, 10 \times (n-1)+10)\}.$$

For example, I^{25} contains agents $i \in \{241, 242, \ldots, 250\}$. Suppose for each agent $i \in I^n$,

$$V_i(\mathbb{W}) = 20 \text{ if } \mathbb{W} = \mathbb{W}^{n,\text{low}} \equiv \{10 \times (n-1)+1, \ldots, 10 \times (n-1)+5\}$$
$$V_i(\mathbb{W}) = 25 \text{ if } \mathbb{W} = \mathbb{W}^{n,\text{high}} \equiv \{10 \times (n-1)+6, \ldots, 10 \times (n-1)+10\}$$
$$V_i(\mathbb{W}) = 0 \text{ if } \mathbb{W} \neq \mathbb{W}^{n,\text{low}} \text{ or } \mathbb{W}^{n,\text{high}}$$

That is, for each coalition of ten adjacent agents, each agent in the coalition gets 20 units of utility if he subscribes to the first five adjacent "low" projects starting from $w = 10 \times (n-1)+1$, and *25* units of utility if he subscribes to the second five adjacent "high" projects starting from $w = 10 \times (n-1)+6$, and zero utility otherwise. Also suppose that,

$$\forall\ w \in \mathcal{W},\ \text{Cost}(w) = 20$$
$$\forall\ i \in \mathcal{I},\ \Omega(i) = 15$$
$$\forall\ i \in \mathcal{I},\ \forall\ \mathbb{W} \in \mathcal{P}(\mathcal{W}),\ \text{TC}_i(\mathbb{W}) = \|\mathbb{W}\|.$$

Consider two feasible allocations. The "low" plan is defined as follows:

$$\forall\ i \in \mathcal{I},$$
$$X^{\text{low}}(i) = 5,$$

$$\forall\ n \in \mathbb{N}_+$$
$$\text{TaxPlan}^{\text{low}}(i, w) = 2 \text{ if } i \in I^n \text{ and } w \in \mathbb{W}^{n,\text{low}}$$
$$\text{TaxPlan}^{\text{low}}(i, w) = 0 \text{ if } i \notin I^n \text{ and } w \in \mathbb{W}^{n,\text{low}}$$
$$\text{TaxPlan}^{\text{low}}(i, w) = 0 \text{ if } w \notin \mathbb{W}^{n,\text{low}}$$

and

$$\forall\ n \in \mathbb{N}_+ \text{ and } \forall\ i \in I^n,$$
$$\text{SubMap}^{\text{low}}(i) = \mathbb{W}^{n,\text{low}}$$

In contrast, the "high" plan is defined as follows:

$$\forall\ i \in \mathcal{I},$$
$$X^{high}(i) = 5,$$

$$\forall\ n \in \mathbb{N}_+$$
$$\text{TaxPlan}^{high}(i,w) = 2 \text{ if } i \in I^n \text{ and } w \in \mathbb{W}^{n,high}$$
$$\text{TaxPlan}^{high}(i,w) = 0 \text{ if } i \notin I^n \text{ and } w \in \mathbb{W}^{n,high}$$
$$\text{TaxPlan}^{high}(i,w) = 0 \text{ if } w \notin \mathbb{W}^{n,high}$$

and

$$\forall\ n \in \mathbb{N}_+ \text{ and } \forall\ i \in I^n,$$
$$\text{SubMaphigh}(i) = \mathbb{W}^{n,high}$$

Observe that both plans are feasible since for each ten agent coalition, a total of five projects are subscribed to. The sum of prices for each project is $10 \times 2 = 20$, which equals the cost of each project. Since each agent pays a total of 10 in taxes and has an endowment of 15, they are left with 5 units of private good to consume.

Also observe that the "low" allocation leaves each agent $i \in I^n$ a utility of:

$$\Omega(i) - \sum_{w \in \mathbb{W}^{n,\,low}} \text{TaxPlan}^{low}(i,w) + V_i(\mathbb{W}^{n,low}) - \| \mathbb{W}^{n,low} \| = 15 - 10 + 20 - 5 = 20$$

while the "high" allocation differs only in that $V_i(\mathbb{W}^{n,high}) = 25$, and so leaves agents with a utility of 25. It is easy to see that, not only that the high plan Pareto dominates the low plan, but that the high plan is, in fact, Pareto optimal.

We claim the low allocation is an EFPE for the following price system $\text{Price}^{low} \in \text{PRICE}$, and $\text{ProdPlan}^{low} \in \text{PROD\_PLAN}$

$$\forall\ n \in \mathbb{N}_+,$$
$$\text{Price}^{low}(i,w) = 2 \quad \text{if } i \in I^n \text{ and } w \in \mathbb{W}^{n,low}$$
$$\text{Price}^{low}(i,w) = 0 \quad \text{if } i \notin I^n \text{ and } w \in \mathbb{W}^{n,low}$$
$$\text{Price}^{low}(i,w) = \emptyset \text{ if } w \notin \mathbb{W}^{n,low}$$

and

$$\text{ProdPlan}^{low}(w) = 1 \text{ if } \exists\ n \in \mathbb{N}_+ \text{ such that } w \in \mathbb{W}^{n,low}$$
$$\text{ProdPlan}^{low}(w) = 0 \text{ if } \exists\ n \in \mathbb{N}_+ \text{ such that } w \in \mathbb{W}^{n,low}.$$

To prove this, we show each of the four equilibrium conditions is satisfied.

(1) $\forall\ i \in \mathcal{I}$,

$$X(i) = \Omega(i) - \sum_{w \in \mathcal{W}_i} \text{Price}(i,w) = \Omega(i) - \sum_{w \in \mathcal{W}_i} \text{TaxPlan}(i,w) = 15 - 10 = 5$$

Thus, the low plan is affordable under equilibrium prices.

(2) Following $\text{SubMap}^{\text{low}}$ gives each agent a utility 20. The alternative of choosing to subscribe any other set of projects offered in equilibrium besides $\mathbb{W}^{n,\text{low}}$ gives agents net negative utility. Only the "low" projects are priced and produced, and $V_i(\mathbb{W}) = 0$ if $\mathbb{W} \neq \mathbb{W}^{n,\text{low}}$. Even though for agents $i \in I^n$, $\text{Price}(i,w)\,\text{low} = 0$ for all projects $w \in \mathbb{W}^{n,\text{low}}$ such that $n \neq \bar{n}$, when transaction costs are subtracted, net utility is negative. Thus, all agents choose to follow the subscription map in equilibrium.

(3) $\forall\ w \in \mathcal{W}$,

$$\text{ProdPlan}(w) = 1 \Leftrightarrow \text{Price}(i,w) = 10 = \text{Cost}(w)$$

$$\text{ProdPlan}(w) = 0 \Leftrightarrow \sum_{i \in \mathcal{I}} \text{Price}(i,w) = 0 < 10 = \text{Cost}(w)$$

and so only projects for which prices exactly cover costs are produced.

(4) Finally, consider any $\bar{w} \in \mathcal{W}$ such that $\text{ProdPlan}(\bar{w}) = 0$. Only "high" projects are not produced, and adding any single high project, $\bar{w} \in \mathbb{W}^{n,\text{high}}$ to a "low" set of subscriptions drops the utility an agent receives from subscriptions to zero: $V_i(\mathbb{W}^{n,\text{low}} \cup \bar{w}) = 0$. The marginal willingness to pay for a subscription to a single high project, therefore, is less than zero, since adding it results in a loss for 21 units of utility (accounting for higher transaction costs). Thus, no extension of the price system to a non-produced project, $\text{Price}(i,\bar{w})$, would yield enough revenue to cover its production costs.

We conclude that

$$\text{Price}^{\text{low}}$$
$$\text{ProdPlan}^{\text{low}}$$
$$\{X^{\text{low}}, \text{TaxPlan}^{\text{low}}, \text{SubMap}^{\text{low}}\}$$

is an EFPE that supports a Pareto dominated feasible allocation.

On the other hand, it is easy to check that

$$\text{Price}^{\text{high}}(i,w) = 2 \ \text{ if } i \in I^n \text{ and } w \in \mathbb{W}^{n,\text{high}}$$
$$\text{Price}^{\text{high}}(i,w) = 0 \ \text{ if } i \notin I^n \text{ and } w \in \mathbb{W}^{n,\text{high}}$$
$$\text{Price}^{\text{high}}(i,w) = \emptyset \ \text{ if } w \notin \mathbb{W}^{n,\text{high}}$$

$$\text{ProdPlan}^{\text{high}}(w) = 1 \ \text{ if } \exists\, n \in \mathbb{N}_+ \text{ such that } w \in \mathbb{W}^{n,\text{high}}$$
$$\text{ProdPlan}^{\text{high}}(w) = 0 \ \text{ if } \exists\, n \in \mathbb{N}_+ \text{ such that } w \in \mathbb{W}^{n,\text{high}}$$

and

$$\{X^{\text{high}}, \text{TaxPlan}^{\text{high}}, \text{SubMap}^{\text{high}}\}$$

is also an EFPE, and that it supports a Pareto optimal allocation.

■

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