Public Goods, Bounded Attention Spans and Equilibrium in the Internet Economy

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Introduction

The only justification for thinking about continuum economies is that they are an economically meaningful limit of a large finite economy.

This does not seem to be true in the case of pure public goods economies as they are usually written.

The point of this paper is propose a new way of modeling pure public goods in a large economy that we hope addresses some of this, and to explore the equilibrium and efficient allocations.

We will also draw some conclusions about the shape and nature of the new information economy.

The problem with Muench's and related approaches is that it is difficult to interpret allocations in a public goods economy with a continuum of consumers.

For example, if there is a positive amount of public goods in an allocation, then the ratio of public to private goods consumption for all consumers is infinity. How does one compare two such allocations? After all, infinity is infinity.

On the other hand, it is fundamentally impossible to distinguish between allocations in which the public goods level almost zero. For such allocations the ratio of public to private goods consumption for each consumer could be bounded, or undefined.

A structural incomparability between public and private good quantities is built in.

Our Approach

Something has to give, and in this paper we propose a new approach to a large public goods economy which we argue does provide a reasonable limiting case.

We restrict attention to pure public (non-rival) goods. If there is crowding or diminishing service quality levels with distance, then we are in a local public goods economy, and the existence and core equivalence properties are well understood.

Even national defense falls into this has a degree of crowding in this sense.

Truly pure public goods are mostly in the category of knowledge and intellectual content. These are goods which are now most frequently delivered over the internet, though radio, television, libraries, and social networks certainly play a role as well.

Our Approach: Properties of knowledge goods

1. They are differentiated.

2. Each of the differentiated products is provided in clear finite amounts.

3. Agents with different tastes but consuming a given item often agree on which other items are close substitutes.

4. As the market grows, the number of intellectual products increases, but the number of agents who choose to consume any given piece of content may not increase.

5. agents don't consume an infinite amount of any given item of content and *they don't consume an infinite number of types of content.* (limited attention spans or time.)

Our Approach

Putting this together, we see the limit of a large pure public goods economy as having an infinity of slightly differentiated pieces of intellectual content all directed at relatively small differentiated audiences.

Thus, we propose a model in which average contributions to pure public goods production can be strictly positive and yet no public good is provided or consumed at infinite levels.

Instead the contributions are absorbed by producing finite levels of an infinite number of pure public goods, each consumed by a finite number of agents.

We argue that this closely reflects what we see in today's internet economy.

A countably infinite set of agents:

$$i \in \mathcal{I} \subseteq \mathbb{N}.$$

$\mathcal{I}$ can be proper subset of the natural numbers.

$i \in I \subseteq \mathcal{I}$ means that agent i is in the coalition I which is a (finite or infinite) subset of the set of agents.

One private good denoted x. Suppose that $i \in I$ and I is finite; then:

$x_i \in X_I \in \Re^{\mid I \mid}$

If I is a countably infinite set, then X_I is interpreted as a countabley infinite sequence.

The Model

A countably infinite set of potential pure public projects:

$$w \in \mathcal{W} \equiv \mathbb{N}.$$

$w \in W \subset \mathcal{W}$ means that the public project with index number w is in the set of public projects W which is a subset of the whole set of public potential public projects.

These are discrete. purely non-rival, public projects without Euclidean structure.

The tax cost of producing a public project $w \in \mathcal{W}$ in terms of private good is denoted.

$$t_w \in [0, T].$$

Note this imposes a maximal tax cost of T over the entire set of potential public projects.

The public projects are consumed (or *Subscribed* to) by agents. A *subscription map* is a set valued correspondence (which may be empty for some elements of the domain) denoted:

$$S : \mathcal{I} \to \mathcal{W}.$$

Thus, if $S(i) = W \subseteq \mathcal{W}$, then agent i subscribes to all public projects $w \in W$.

Define the associated *membership map*, $M : \mathcal{W} \to \mathcal{I}$, as:

$$M(w) \equiv \{i \in \mathcal{I} \mid w \in S(i)\}.$$

Thus, if $M(w) = I \subseteq \mathcal{I}$, then project w has a membership of consisting of all agents $i \in I$.

We will generally use the shorthands:

$S(i) \equiv W_i$ to denote the set projects subscribed to by agent i

$M(w) \equiv I_w$ to denote the set of agents who hold a membership for project w.

Given a subscription map S, define the set of projects that are produced as those that have at least one subscriber:

$$\mathcal{W}^S \equiv \{w \in \mathcal{W} \mid \exists\ i \in \mathcal{I}\ s.t.\ w \in S(i)\}.$$

$\omega_i \in \Omega_I$: a vector or sequence that gives the private good endowment for the coalition I.

The Model

Agents have a utility function of the form:

$$u_i(x_i, W_i) = x_i + v_i(W_i) - a_i(W_i).$$

$a_i(W_i)$ as the attention cost of subscribing to the the set W_i of different public projects.

Note that this embeds an assumption there is no "intensity of consumption" decision. The motivation for this is that there a search, attention, or transaction cost of calling up any given set of web pages, getting a set of books off the shelf, putting on at set of CD's, and so on.

Assumption 1: For all $i \in I$, $v_i(\emptyset) = 0$ and $a_i(\emptyset) = 0$.

Assumption 2: There exists a finite bound $\bar{u} > 0$ such that for all $i \in \mathcal{I}$ and all possible subscription choices (finite or infinite), $v_i(W_i) \leq \bar{u}$

Assumption 3: There exists $B \in \mathbb{N}$, such that for all $i \in I$ if $\mid W_i \mid \; > B$, then $a_i(W_i) > \bar{u}$

Assumption 4: There exists $\epsilon > 0$ and $\delta \in (0, 1]$ such that for all subscription maps S and all projects $w \in \mathcal{W}$ such that $I_w \neq \emptyset$:

a. if I_w finite, there exists a subcoaltion of agents $\hat{I} \subseteq I_w$ such that $|\,\hat{I}\,| \geq \mathrm{INT}(\delta \times |\,I_w\,|)$ (where $\mathrm{INT}(z)$ is the largest integer weakly less than z), and an alternative project $\hat{w} \in \mathcal{W}$ where for all $i \in \hat{I}$, $\hat{w} \notin W_i$ such that:

$$u_i(x, W_i) = x_i + v_i(W_i) - a_i(W_i) < x_i + v_i(W_i \cup \hat{w} \setminus w) - a_i(W_i \cup \hat{w} \setminus w) - \epsilon$$

b. if I_w countable infinite, there exists a countabley infinite subcoaltion of agents $\hat{I} \subseteq I_w$, and an alternative project $\hat{w} \in \mathcal{W}$ where for all $i \in \hat{I}$, $\hat{w} \notin W_i$ such that:

$$u_i(x, W_i) = x_i + v_i(W_i) - a_i(W_i) < x_i + v_i(W_i \cup \hat{w} \setminus w) - a_i(W_i \cup \hat{w} \setminus w) - \epsilon.$$

Assumption 1 is just a normalization that says if agents do not subscribe to any public goods, they receive no consumption benefits and pay no attention costs.

Assumption 2 says that there is an upper limit on the utility that agents can get from any set of subscriptions. To allow otherwise would be to imagine that one either achieves Nirvana while consuming a finite set of goods, or can approach it as one consumes public goods without bound.

Assumption 3 says that at some point, the attention cost of consuming one more webpage exceeds any possible gain. We call this the “go to bed” constraint

Assumption 4 is a weak way of capturing the idea of the existence of close substitutes for any public project. Specifically, the assumption says the following: consider any subscription system S and set of agents consuming any given public project w. There will exist $\hat{w}$ which is a close substitute for w in the following sense: We can always select a group $\hat{I}$ from the set of agents consuming the good w, who are also not currently subscribing to $\hat{w}$, such that this group is at least a fraction δ as big as I_w such that these agents prefer an allocation in which $\bar{w}$ has been exchanged for w by at least ϵ private good. Since δ can be very small, Assumption 4 will only bite in general if a very large number of agents are consuming a given project.

Defining a feasible allocation is a little bit tricky in this environment since the society has access to a infinite quantity of private goods and will in general spend an infinite amount to produce an infinite number of public projects. Since all such infinities are equivalent, it would not be economically meaningful to show that the sum of tax expenditures equaled the sum of tax collections from agents.

The natural approach would be to require that some sort of average cost of providing subscriptions to agents equaled an average contribution by agents to public project production. Given a private goods allocation $X_{\mathcal{I}}$, we can see that each agent $i \in \mathcal{I}$ is implicitly contributing $(\omega_i - x_i)$ to public project construction. One might therefore consider the Cesàro mean of the sequence $\{\omega_i - x_i\}$ which is defined as:

$$lim_{n \to \infty} \sum_{i=1}^{n} \frac{(\omega_i - x_i)}{n}.$$

Unfortunately, even if the contributions are taken from a compact set, this Cesàro mean may not exist. It could be that the sequence takes increasingly long oscillations between high and low values and so the mean also oscillates as you go systematically farther out in the sequence.

We propose instead form a new sequence of arbitrary but finite length $n \in \mathbb{N}$ by randomly sampling with uniform probability the original uncountably infinite sequence. Since this randomly reorders the sequence's elements, any pattern of oscillations that prevented Cesàro mean the would disappear. Thus, by the law of large numbers, the Cesàro mean will exist for the sampled sequence.

More formally, consider any countabley infinite set of index numbers $N \subseteq \mathbb{N}$. Let N^n be defined as a finite subset of N which is constructed by randomly drawing n elements with uniform probability from N. Thus, $N^n, \hat{N}^n, \bar{N}^n$ denote different random selections of n elements from the parent set, N.

Our Approach

We use this approach to find the average cost of providing subscriptions to agents under any given subscription mapping S as follows. Begin by taking a random sample of n agent names from $\mathcal{I}$, and a random sample of n project names from $\mathcal{W}$. Note, of course, that it will not be the case in general that all agent names correspond to agents actually in the population, nor that all the project names drawn in the sample are in fact produced.

Partition $\mathcal{I}$ as follows:

$$\mathcal{I} \equiv \mathcal{I}^F \bigcup \mathcal{I}^C$$

where $\mathcal{I}^F$ is the set of agents $i \in \mathcal{I}$ such that W_i is finite, $\mathcal{I}^C$ is the set of agents $i \in I^n$ such that W_i, is countabley infinite.

Given the sample, define the set of agents in the sample who hold a finite or countabley infinite set of memberships respectively as:

$$I^{Fn} \equiv \mathcal{I}^{F} \bigcap I^{n}, \text{ and } \ I^{Cn} \equiv \mathcal{I}^{C} \bigcap I^{n}.$$

Our Approach

Partition $\mathcal{W}^S$ first into two subsets, $\mathcal{W}^F$ and $\mathcal{W}^C$ consisting of projects that have finite memberships and countably infinite memberships, respectively, under S, and then into four subsets as follows:

$$\mathcal{W}^{FF} = \mathcal{W}^F \bigcap \{w \in \mathcal{W}^S \mid \exists\ i \in I_w \text{ and } W_i \text{ is finite }\}$$

$$\mathcal{W}^{FC} = \mathcal{W}^F \bigcap \{w \in \mathcal{W}^S \mid \forall\ i \in I_w \text{ and } W_i \text{ is countably infinite }\}$$

$$\mathcal{W}^{CF} = \mathcal{W}^C \bigcap \{w \in \mathcal{W}^S \mid \forall\ i \in I_w \text{ and } W_i \text{ is finite }\}$$

$$\mathcal{W}^{CC} = \mathcal{W}^C \bigcap \{w \in \mathcal{W}^S \mid \exists\ i \in I_w \text{ and } W_i \text{ is countably infinite }\}$$

Our Approach

Finally, we are ready to figure out the average cost of producing the cost the projects in $\mathcal{W}^S$ using our sampling technique. Our strategy is to define an artificial apportioning of the cost of producing each of these subsets of $\mathcal{W}^S$ over agents for any given sample in such a way that the costs will be exactly covered. Given this feasible apportioning, we can estimate the average cost of public good production with arbitrary precision by choosing a large enough sample. Given this, we use a similar technique to estimate the average contribution of agents to public goods production under any given allocation. If these two averages are the same, then we call the allocation feasible.

$\mathcal{W}^{FF}$: Consider any agent $i \in I^{Fn}$. Partition the finite set of subscriptions he holds as follows: $W_i \equiv W_i^F \bigcup W_i^C$ where $W_i^F \subseteq \mathcal{W}^F$ and $W_i^C \subseteq \mathcal{W}^C$. Now consider any project $w \in W_i^F$ which has a finite membership consisting of I_w. Apportion the costs of such projects equally over the subscribers who have only a finite set of subscriptions in total. This allows any subscribers who happen to have a countably infinite set of subscriptions in total to free ride. Under this rule, it costs $t_w / \mid I_w \bigcap \mathcal{I}^F \mid$ to provide this subscription to agent i. Thus, the average cost of providing agents in I^n with subscriptions to projects in $\mathcal{W}^{FF}$ is:

$$\frac{1}{\mid I^{Fn} \mid} \sum_{i \in I^{Fn}} \sum_{w \in W_i^F} \frac{t_w}{\mid I_w \bigcap \mathcal{I}^F \mid}.$$

$\mathcal{W}^{CC}$: Let $W^{CCn} \subseteq W^{Cn}$ be the subset of the projects in the sample that have countably infinite memberships, at least some of which are held by agents in I^C. Apportion all the costs of such projects equally over agents who happen to have a countabley infinite set of subscriptions in total. This allows any subscribers who happen have only a finite set of subscriptions in total to free ride.

To figure out the average cost of providing such subscriptions, we need a sense of the proportion of agents to projects. We will respect the counting metric to do so. Thus, using our sample of the project space of size n, define $I^{CCn} \subseteq I^{Cn}$ to be the subset of agents in the sample who have a countabley infinite number of subscriptions in projects with a countably infinite membership. Thus, $\mid I^{CCn} \mid /n$ is the fraction of agents who have a countabley infinite number of subscriptions to such projects in our sample. It follows that $\mid W^{CCn} \mid / \mid I^{CCn} \mid$ is the rate at which such projects increase relative to relevant population. Using our apportioning rule, the average cost of providing agents in I^n memberships for projects in $\mathcal{W}^{CC}$ is:

$$\frac{\mid W^{CCn} \mid}{\mid I^{CCn} \mid} \sum_{w \in W^{CCn}} \frac{t_w}{\mid W^{CCn} \mid} = \sum_{w \in W^{CCn}} \frac{t_w}{\mid I^{CCn} \mid}.$$

Putting this together, the average cost of providing agents in I^n all of their subscriptions is:

$$\frac{1}{\mid I^{Fn} \mid} \sum_{i \in I^{Fn}} \sum_{w \in W_i^F} \frac{t_w}{\mid I_w \bigcap \mathcal{I}^F \mid} + \sum_{w \in W^{CCn}} \frac{t_w}{\mid I^{CCn} \mid} + \sum_{w \in W^{FCn}} \frac{t_w}{\mid I^{FCn} \mid}.$$

Given this, a *feasible allocation* is a private good allocation $X_{\mathcal{I}}$ and a subscription mapping S such that for all $\epsilon > 0$ and probabilities $p \in (0, 1]$ there exists $n \in \mathbb{N}$ such that for any random sample from $\mathcal{W}$ and $\mathcal{I}$ of size n, the probability that the following inequality will be satisfied is at least p:

$$\sum_{i \in I^n \bigcap \mathcal{I}} \frac{(\omega_i - x_i)}{n} \geq \frac{1}{\mid I^{Fn} \mid} \sum_{i \in I^{Fn}} \sum_{w \in W_i^F} \frac{t_w}{\mid I_w \bigcap \mathcal{I}^F \mid} + \sum_{w \in W^{CCn}} \frac{t_w}{\mid I^{CCn} \mid} + \sum_{w \in W^{FCn}} \frac{t_w}{\mid I^{FCn} \mid} + \epsilon.$$

A feasible allocation X, S is ϵ-*Pareto Optimal* if there does not exist another feasible allocation $\hat{X}, \hat{S}$ such that for all $i \in \mathcal{I}$:

$$u_i(\hat{x}_i, \hat{W}_i) > u_i(x_i, W_i) + \epsilon.$$

A feasible allocation X, S is *Pareto Optimal* if it is ϵ-*Pareto Optimal* for $\epsilon = 0$.

Some Results

Lemma 1. *There is an upper bound on how many subscriptions any agent will have in any PO allocation X, S.*

Lemma 2. *There is a finite upper bound on how many agents will subscribe to any given public good in any PO allocation X, S.*

Put together, Lemmas 1 and 2 say that the equilibria of an economy that satisfies assumptions 1 through 4 will have the properties outlined in the introduction.

Theorem 1. *For all $\epsilon > 0$, there exists an $\epsilon-$Pareto optimal allocation.*

Equilibrium

Agents are price takers. The price system consists of infinite sequence that gives a price to buy a subscription in every potential project in $\mathcal{W}$. Agents maximize their utility function by choosing a finite subset (given Lemma 1) from the countable infinity of projects offered to add to his subscription list.

$$\mathcal{P} = \{p_1, p_w, \ldots, p_w, \ldots\}$$

where p_w is the subscription price that each agent must pay for project w. We will require for all $w \in \mathcal{W},\ p_w \in [0, 2\bar{u}]$ These prices might represent the price of a book or CD. In the case of electronic public goods like websites, they might represent subscription fees, or more likely, the cost imposed on agents having to look at banner-adds or pop-ups.

Our Approach

In the interests of simplicity, we will assume that each public project can be produced by one specific firm. A firm's demand conjecture is a mapping denoted:

$$N^w : [0, 2\bar{u}] \to \mathbb{N}$$

where $N^w(p)$ is the number of subscriptions the firm producing good w expects to see at anonymous price p.

Since firms may make profits in equilibrium and these must be returned to agents in equilibrium. We will follow the convention that all agents own equal shares of all firms in the interest of simplicity. We can use our sampling technique to estimate these by observing that the difference between the average contribution from each agent and the average cost of a public project weighted by the ration of agents to projects overall given the average profit per agent:

$$\pi^n \equiv \sum_{i \in I^n \bigcap \mathcal{I}} \frac{(\omega_i - x_i)}{\mid I^n \bigcap \mathcal{I} \mid} - \frac{\mid W^n \bigcap \mathcal{W}^S \mid}{\mid I^n \bigcap \mathcal{I} \mid} \left(\sum_{w \in W^n \bigcap \mathcal{W}^S} \frac{t_w}{\mid W^n \bigcap \mathcal{W}^S \mid} \right) =$$

$$\frac{1}{\mid I^n \bigcap \mathcal{I} \mid} \left(\sum_{i \in I^n \bigcap \mathcal{I}} (\omega_i - x_i) - \sum_{w \in W^n \bigcap \mathcal{W}^S} t_w \right).$$

Given this, feasible allocation X, S and a price and demand conjecture system $\mathcal{P}, N$ for endowments Ω is an *Anonymous Equilibrium* if such that for all $\epsilon > 0$ and probability $p \in (0, 1]$ there exists $n \in \mathbb{N}$ such that for any random sample from $\mathcal{W}$ and $\mathcal{I}$ of size n, the probability that the following conditions will be satisfied is at least p:

1. For all $i \in \mathcal{I}$, and all $\hat{W}_i \subseteq \mathcal{W}$:

$$\omega_i + v_i(W_i) - a_i(W_i) - \sum_{w \in W_i} p_w + \pi^n \geq$$

$$\omega_i + v_i(\hat{W}_i) - a_i(\hat{W}_i) - \sum_{w \in \hat{W}_i} p_w + \pi^n + \epsilon$$

2. For all $w \in \mathcal{W}^S$

$$p_w \mid I_w \mid - t_w = p_w N^w(p_w) - t_w \geq 0$$

3. For all $w \in \mathcal{W}^S$ and all $\hat{p} \in [0, 2\bar{u}]$ such that $\hat{p} \neq p_w$:

$$\hat{p} N^w(\hat{p}) - t_w \leq 0$$

4. For all $\hat{w} \notin W^S$ and all $\hat{p} \in [0, 2\bar{u}]$ it holds that

$$\hat{p} N^w(\hat{p}) - t_w \leq 0$$

Condition 1 says that taking the subscription prices as given, almost all agents choose an affordable, utility maximizing set of projects.

Condition 2 says that for almost all projects produced in the equilibrium, taking price and demand as given, costs are at least covered.

Condition 3 says that given each firm's demand speculations, no public project which is not produced could generate positive profits at equilibrium prices.

Condition 4 says that for almost all projects produced in equilibrium, the equilibrium demand system N posited by producers agrees with the actual subscriptions demanded at the equilibrium price at least for public projects that have finite membership.

Remark 1. Suppose agents 1 and 2 get utility level 60 and 70 respectively from public project 1, and 80 and 90, respectively from project 2. Both cost 100, and the agents don't care about any other projects. If attention costs are low enough the only PO allocation would have the agent 1 and 2 form jointly consume project 2.

Suppose we set at $p_1 = 50$ and $p_2 = 200$. Then no agent would find it optimal to consume the second good in equilibrium and demand would correctly projected at zero under these prices. Firm 2 could project that $N^2(p) = 0$ for all p and would therefore choose not to produce. Thus, a Pareto dominated allocation is an equilibrium.
(To complete the economy, assume, agent 3 and 4 have the same preferences for projects 3 and 4, and so on.)

Remark 2.Suppose agent 1 and 2 get utility level 40 and 80 respectively from public project 1 which costs 100 and don't care about any other projects. If attention costs are low enough the only PO allocation would have the agent 1 and 2 form jointly consume project 1.

If $p_1 \leq 40$, then both agents demand the good, but costs are not covered
If $40 < p_1 \leq 80$, then only agent 2 demands the good and also costs are not covered
If $40 < p_1 \leq 80$, then no agent bys the good.
Thus, no anonymous prices can support the PO.
(Again to complete the economy, assume agents 3 and 4 have the same preferences for project 3, and so on.)

These counterexamples gives us the following result:

Theorem 2. *There may exist anonymous equilibria which are not Pareto optimal.*

Theorem 3. *It is not possible to decentralize every Pareto optimal allocation for some set of anonymous prices and initial endowments.*

That is both the First and Second Welfare Theorems fail.

Désolé!

Our Approach

More stuff on non-anonymous equilibrium and a Second Welfare Theorem under strong conditions goes here........

Conclusions

In this paper, we were motivated by two concerns.

First, we wanted to provide a model of a public goods economy with an infinite number of consumers that was an economically and mathematically meaningful limit of a large finite public goods economy.

Second, we wanted to provide a positive analysis of the properties of such an economy based as much as possible on the institutional details and constraints we observed in the real world.

Conclusions

We argued that it was unlikely that the levels of public goods consumed by agents would grow without bound or go to zero as an economy One of these seem necessary in large or continuum economies as currently written.

We proposed an alternative: as the economy gets large, the number of public goods (which we think of as internet or information goods) also gets large.

Conclusions

We showed that under fairly mild conditions the Pareto optimal allocations of this economy will involve an infinite number of public projects being produced and that each of these projects being consumed by a finite number of agents.In addition, each agent would only consume a finite number of public projects.

We also showed that although the FWT and SWT fail in general for anonymous equilibrium.

Conclusions

- Equilibrium price systems have extremely (and unrealistically) high information requirements. Even then, a great deal which is not PO can arise as an equilibrium.
- Thus,we should probably expect that we are not at a first best outcome in the information economy.
- Entrepreneurs take educated guesses about what will succeed but there are no arbitrage opportunities implied by disparities in the cost/revenue signals from equilibrium price system that are visible to all.
- It is possible to get rich (that is, make economic profits) if you happen to stumble on a public project that you can produce cheaply and is in high demand. It is not at all surprising that no one beat you to it. There may indeed be five dollar bills laying on the ground in the new information economy.