Optimal and Equilibrium Membership†
in Clubs in the Presence of Spillovers

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Abstract

This paper treats a partial equilibrium club economy in which clubs impose positive or negative externalities on one another. Both spillovers in congestable good and crowding are studied. It is shown that an increase the relative strength of the first type of spillover may result in either an increase or decrease in the optimal club size and public good levels. Spillovers in crowding, on the other hand, have a definite effect on the optimal membership and production of public good. These optimal outcomes are then compared to Nash equilibrium club size and public good provision when clubs are established by profit maximizing entrepreneurs instead of a social planer.

1. Introduction

Over thirty years ago, Buchanan [5] introduced the notion of a club good. By this he meant a good whose consumption was subject to partial rivalry. His paper was one of the first attempts to make rigorous the model informally presented by Tiebout [15] in his seminal work on local public goods. The main difference between these two approaches is that Tiebout had in mind a general equilibrium model in which agents choose the coalition with whom they will share the cost of public goods by "voting with their feet." That is, express their preferences by physically moving to a location. In this context it is most natural to imagine agents partitioning themselves into disjoint jurisdictions by joining one and only one coalition. In contrast, Buchanan's club model takes a more partial equilibrium view of the world. He imagines competitive clubs which are set up by price taking, profit maximizing entrepreneurs for the purpose of providing a particular type of public good or service (country clubs or private schools, for example). Since these clubs are not tied to a notion of location, it is natural to allow agents to join several clubs, or even no clubs at all. There is now an extensive literature on clubs and club goods. Excellent surveys are provided in Sandler and Tschirhart [14] and Cornes and Sandler [6].

We take Buchanan's partial equilibrium approach in this paper. Our focus is to study how positive and negative spillovers affect the optimal and equilibrium size and public goods provision in clubs. Clearly spillovers are an important real world phenomenon. For example, sewage treatment of an upstream city benefits downstream cities; radio and TV broadcasts in one country can be seen in bordering jurisdictions. Vigorous crime prevention in one city could either lower regional crime, or force crime into neighboring communities. Museums and parks are visited and, therefore, crowded by agents from nearby jurisdictions. Nevertheless, Buchanan's characterization of optimal clubs ignores the possibility of spillovers, and Tiebout explicitly excludes them from his model.

There is a small literature that treats spillovers. In particular, see Williams [17], Brainard and Dolbaer [4], Pauly [7], Boskin [3], Sandler [10], Sandler and Cauley [11], Sandler and Cuyler [12, 13] and more recently, Wellisch [16].

The main difference between our model and the models mentioned above is that the previous literature assumes a fixed number of jurisdictions and does not treat crowding (and the corresponding externality that comes with it) explicitly . With a fixed number of jurisdictions, it is not interesting to examine questions regarding to optimal jurisdiction size. As we mentioned in the beginning, Tiebout envisioned an economy where the number of jurisdictions is determined endogenously; each agent chooses the jurisdiction that best suits his preferences for the mix of private and public goods and community size. We study a Buchanan model in which club entrepreneurs offer an entire package of private good, club good, and congestion and agents choose the club they most prefer. Hence, we incorporate crowding as an explicit variable in the utility of the agents. This permits us to examine externalities from crowding; that is, agents from neighboring jurisdictions enjoy part or all of the public good produced in our jurisdiction, without paying for it.

More formally, we investigate a one private good, one public good, quasi-linear economy with identical agents. Each agent has an endowment of the private good, and we assume that the technology to transform private into public goods is linear. We also assume that each club or jurisdiction is affected by the characteristics of k other clubs, where k is exogenously fixed. This is meant to reflect an economy in which spillovers come from neighboring jurisdictions rather than being felt economy wide. For example, people in Hoboken benefit from New York City's investment in the arts, but people in Chicago are largely unaffected.

We explore two types of spillovers. The first are spillovers of local public goods with no externalities in crowding. Examples of this include mosquito eradication programs, and radio and TV broadcasts. In effect, a fraction of the public good produced in one club is experienced by surrounding clubs, and is a perfect substitute for their own provision of public goods. The second type of spillover is a consequence not from the public goods provision, but from the crowding of facilities by agents from neighboring clubs. Typical examples are museums, parks and zoos.

One might suspect that in the presence of positive externalities between localities it is efficient to consolidate the economy into a small number of more populous clubs producing larger levels of public goods. This would seem to result in an improvement in efficiency by partially internalizing the positive externality. Our first set of results, however, show that this need not be the case. When the spillover strength changes, the optimal public good production and the optimal jurisdiction size may increase or decrease. This result is quite robust. It holds for both positive and negative spillovers, and for spillovers resulting both from public goods production and crowding.

We also explore the Nash equilibrium public goods levels and populations of clubs. We describe a model in which profit maximizing entrepreneurs establish clubs and compete to attract members by offering various combinations of public goods levels, club size and admission fees. Of course, these entrepreneurs take the actions of their fellows as given. As in the social planning problem, we find that the effect of the spillover strength is ambiguous in all cases. We show, however, that the Nash equilibrium levels of public goods production and club size are higher than the optimal levels in the presence of negative spillovers and lower in the presence of positive spillovers.

In the next section, we formally define the model. In section three we explore the effect of spillovers on the optimal club size and provision of public good. In section four we characterize the Nash equilibrium outcome and compare this to the efficient outcomes. In section five we show a simple tax/subsidy scheme which restores efficiency in the Nash equilibrium case. Section six closes the paper with some concluding remarks.

2. The Model

We consider a one private good, one public good economy. All consumers are identical and have preferences which are represented by the utility function:

$$U(x, y, n) = x + u(y) + v(n)$$

where x is the level of transferable private good, y is the level public good provided by the club, and n is the size of the membership of the club the agent joins.[1] We make the standard convexity assumptions on the utility function: $u' > 0$, $u'' < 0$ and $v' < 0$, $v'' < 0$. Each agent has an endowment w of the private good. We assume that the technology to produce the public good is linear, and $\alpha > 0$ is the marginal cost of public good in terms of private goods. For simplicity, we focus on the case in which each agent joins one club only. It will be obvious how the analysis in this paper can be extended allowing agents to join several clubs (perhaps one for each of several different public goods, as Buchanan imagined), provided that the utility function remains separable.

We assume that each club is affected positively or negatively by actions of k neighboring clubs. The parameter k is assumed to be fixed and exogenous. We think of k typically being in the range of small integers, perhaps, $k = 1, 2, 3$. For example, a county has three or four neighboring counties, or a city may have a few surrounding cities (or suburbs) which are affected by the public good. Thus, we are modeling local spillovers rather than economy-wide externalities like CO_2 production.

Note that we take a very simple approach to local spillovers in this paper. In particular, we do not specifically treat land or physical location in our model. We assume that all agents who live in a given community are subjected to the identical levels of spillovers from neighboring communities. There is no sense in which a agent might be closer or further away from the boarder and thus experience greater or lesser degrees of the spillover. We also assume that the spillover drops to zero for non-neighboring communities. For many types of externalities (smoke, for example) a more realistic approach would be to add physical location to the model and let the spillover decrease as a function of distance. This might yield some interesting comparative statics with respect to land prices. Doing so, however, would substantially complicate the model and probably would not change the qualitative results we obtain on optimal and equilibrium community size and public goods levels. Thus, while this would be an

[1] Note that we make the assumption of separability. This is done to simplify the analysis, since otherwise we would need to sign the cross-partial derivatives, in particular between y and n.

interesting way to extend research on spillovers, we choose to keep the current model simple in order to focus on the questions that are of concern here.

We consider two types of spillovers. The first type results from the public good produced in one club and being partially experienced by other clubs. This external effect may be positive or negative. For example, higher spending of law enforcement in Chicago takes criminals off the street and may lower crime in all the surrounding suburbs. In effect, a fraction of Chicago's law enforcement spills over into the suburbs and is added to whatever law enforcement efforts the suburbs provide. Alternatively, high spending in Chicago may force criminals out of the city and into the suburbs in hopes of finding easier targets. In this case a fraction of Chicago's law enforcement effort spills over into the suburbs and subtracts from the efforts the suburbs choose for themselves.

We also consider the possibility that spillovers take place in crowding. For example, when a city increases its population, its citizens may use the roads of neighboring cities more intensively. Thus, a fraction of the population of any given city spills over and crowds the public facilities of nearby cities. The effect of a larger population could be positive as well. As a city grows in size, more businesses and restaurants may open locally which would decrease the desire of the residents to crowd the facilities of neighboring cities.

Formally, the parameters θ_p and θ_c will denote the strength of the spillovers in public goods and crowding, respectively. We interpret these parameters as the fraction of public good and population chosen by any given jurisdiction that is experienced by neighboring jurisdictions. Economically, it makes the most sense to constrain these θ's to lie in the interval $[-1, 1]$.[2] A value of one would mean that public good and population in one jurisdiction are felt at full strength by neighboring jurisdictions. This would mean that the public good is pure in the sense that jurisdictional boundaries do not impede consumption and that the boundaries are in some sense artificial since

[2] This is not necessary for the analysis, but there are problems in interpretation when we go outside these bounds. For example if θ_p where greater than one, this would mean that if one club broadcasts an hour's worth of radio, the surrounding clubs would hear more than one hour.

you are crowded identically by agents regardless of where they claim to live. A value of negative one would mean that public good is zero sum. For example, if there is a certain amount of garbage that must be dumped in some jurisdiction, one jurisdiction's effort to collect and dump the garbage into the next jurisdiction is completely offset on a one-to-one basis by the next jurisdictions effort to reverse the direction of dumping. It is not clear that minus one has a special interpretation in the case of population. Obviously, if the θ's are zero, there are no externalities between jurisdictions.

Note that when θ_p is positive, clubs benefit from their neighbors, but when θ_c is positive, clubs are damaged by their neighbors (since a positive spillover of crowding is a negative externality). Thus, it is not immediate whether to refer to a positive θ_c as a positive or negative spillover. In this paper we will choose our nomenclature based on economic rather than mathematical considerations. Therefore a positive θ_p and a negative θ_c will be called *positive* spillovers since they are both beneficial.

Exactly how much public good and crowding an agent experiences depends on the actions of the k surrounding communities. At a symmetric equilibrium in which all clubs produce y public goods, and have population n the total public good levels and crowding experienced by agents would be

$$y + \theta_p k y \quad \text{and} \quad n + \theta_c k n,$$

respectively.

3. The Social Planner's Problem

For efficiency in this economy, the social planner maximizes the utility function of the representative agent, subject to his resource constraint:

$$\max_{x,y,n} U(x,y,n) = x + u(y + \theta_p k y) + v(n + \theta_c k n) \ \text{ s.t. } \ x + \frac{\alpha y}{n} = w. \tag{1}$$

Note that given our convexity assumptions, the social optimum must be symmetric. Also, because of quasilinearity, the Lagrange multiplier of this problem is strictly positive, so that we can substitute x for its expression from the budget constraint. The problem is then:

$$\max_{y,n} \quad U(y,n) = w - \frac{\alpha y}{n} + u(y + \theta_p k y) + v(n + \theta_c k n). \tag{2}$$

The first order conditions of the unconstrained problem (2) are:

$$nu' + n\theta_p k u' = \alpha \tag{3}$$

$$-nv' - n\theta_c k v' = \frac{\alpha y}{n} \tag{4}$$

We assume the Hessian matrix to be negative definite[3] and so the sufficient second order conditions for a maximum are satisfied. Denote the solution:

$$(\bar{y}, \bar{n})_{SP}$$

where the subscript SP stands for "Social Planner".

Equation (3) is the Samuelson [8, 9] condition for clubs with spillovers. Note that when $\theta_p = 0$ it becomes the Samuelson condition of the Buchanan model, as discussed in Berglas [1] and Boadway [2]. The first term on the left-hand side of (3) is the sum of the marginal utilities of all the members of the jurisdiction; the second term shows the sum of the marginal utilities of the consumption of the public good weighted by the total spillover effect (i.e., the intensity θ_p of the spillover times the number of the jurisdictions that affect our own jurisdiction). This term may be positive or negative, depending on whether the spillover is positive or negative (that is, whether θ_p is positive

[3] For the Hessian matrix to be negative definite the principal minors have to be of alternating sign. In our model this means that (see equation (6) below) that the term $(1 + \theta_p k)^2 u'' < 0$ and that for the Hessian determinant the following must be true:

$$n^4 (1 + \theta_p \ k)^2 (1 + \theta_c k) u'' v'' - 2\alpha y n (1 + \theta_p \ k)^2 u'' > \alpha^2$$

This last condition will be satisfied if spillovers in crowding are not too big and the public good is sufficiently cheap to produce.

or negative). Note that since the marginal utility of the private good is 1, u' is also the marginal rate of substitution of the public for the private good (MRS_{yx}). Equation (4) is the condition for optimal membership size. If $\theta_c = 0$ the total marginal utility of crowding in the jurisdiction has to equal the average cost of the public good. As there are spillovers in crowding, condition (4) reflects this fact by including the disutility caused by the share of people coming from the k surrounding jurisdictions. Again, since the marginal utility of the private good is 1, v' represents as well the marginal rate of substitution of the membership size for the private good, MRS_{nx}.

We want to focus our attention on the effect on the public good production and jurisdiction size when the spillover effects θ change. In particular, we expect that as the spillover effect increases, it should always be optimal to decrease the number of jurisdictions in the economy to internalize the effect of the spillovers. In other words, consolidating the economy into a fewer number of jurisdictions and therefore partially internalizing the externality seems to be optimal. But this will not necessarily be the case, as we are going to show below. Before proceeding with the analysis, it is convenient to define the elasticity of the marginal utility with respect to public good consumption as the following expression: [4]

$$\varepsilon_{u'} = -u'' \ (1 + \theta_p \ k)^2 \frac{y}{u' \ (1 + \theta_p \ k)} \tag{5}$$

This measures the responsiveness of the marginal valuation of the public good when one more unit of the good is consumed.

Next, we show the following propositions.

Proposition 1. *The optimal output of public good y and the optimal jurisdiction size n will increase in response to an increase in the spillover parameter θ_p if $\varepsilon_{u'} < 1$. Conversely, the optimal output of public good y and the optimal jurisdiction size n will decrease in response to an decrease in the spillover parameter θ_p if $\varepsilon_{u'} > 1$.*

Proof/

[4] This is a definition, $\frac{\partial MU}{\partial y} \frac{y}{MU}$. The term $(1 + \theta_p \ k)$ obviously cancels. The elasticity is written out completely for the purpose of clarity.

Totally differentiating equations (3) and (4) at the optimum $(\bar{y}, \bar{n})_{SP}$ we get:

$$\begin{bmatrix} (1+\theta_p k)^2 u'' & \frac{\alpha}{\bar{n}^2} \\ \frac{\alpha}{\bar{n}^2} & -\left(\frac{2\alpha\bar{y}}{\bar{n}^3}\right) + (1+\theta_c k)v'' \end{bmatrix} \begin{bmatrix} \frac{d\bar{y}}{d\theta} \\ \frac{d\bar{n}}{d\theta} \end{bmatrix} =$$

$$\begin{bmatrix} -k[u' + \bar{y}(1+k\theta_p)u''] & 0 \\ 0 & -k[v' + \bar{n}(1+k\theta_c)v''] \end{bmatrix} \tag{6}$$

Assume that the spillovers in crowding remain constant. We have only a change in the spillovers of the public good. From this, by Cramer's rule, we get the following result for $\frac{d\bar{y}}{d\theta_p}$ and $\frac{d\bar{n}}{d\theta_p}$:

$$\frac{d\bar{y}}{d\theta_p} = \frac{k[u' + \bar{y}(1+k\theta_p)u'']\left[\left(\frac{2\alpha y}{\bar{n}^3}\right) - (1+\theta_c k)v''\right]}{|H|} \tag{7}$$

$$\frac{d\bar{n}}{d\theta_p} = \frac{\frac{\alpha k}{\bar{n}^2}[u' + \bar{y}(1+k\theta_p)u'']}{|H|} \tag{8}$$

where $|H|$ is the Hessian determinant (the determinant of the matrix on the left hand side of equation (6) above).

Note that in (7) and (8) the terms $\left[\left(\frac{2\alpha y}{\bar{n}^3}\right) - v''(1+\theta_c k)\right]$ and $\frac{\alpha k}{\bar{n}^2}$ are positive. This means that those equations depend on the sign of

$$u' + \bar{y}(1+k\theta_p)u'' \tag{9}$$

Suppose we want (7) and (8) to be positive. Then, for this to hold, it must be true that

$$u' + \bar{y}(1+k\theta_p)u'' > 0$$

which, when rearranged, is

$$\varepsilon_{u'} < 1 \tag{10}$$

The left hand side of (10) is the above mentioned elasticity of the marginal utility (which is also the marginal rate of substitution) with respect to the public good. If this elasticity is less than 1, it is optimal to have a higher output of y and bigger jurisdiction sizes n as the spillover strength increases. **QED**

Proposition 2. *The optimal level of public good and jurisdiction size always decrease as θ_c increases. The optimal level of public good and jurisdiction size always increase as θ_c decreases.*

Proof/

Assume the spillovers in the public good fixed. From (6) above, we obtain the following expressions for $\frac{d\bar{y}}{d\theta_c}$ and $\frac{d\bar{n}}{d\theta_c}$:

$$\frac{d\bar{y}}{d\theta_c} = \frac{\frac{k\alpha}{n^2}\left[(1+\theta_c k)v''\bar{n} + v'\right]}{|H|} \tag{12}$$

$$\frac{d\bar{n}}{d\theta_c} = \frac{-k\left[(1+\theta_p k)^2 u''\right]\left[\bar{n}(1+\theta_c k)v'' + v'\right]}{|H|} \tag{13}$$

By assumption the Hessian determinant is positive; hence we have to sign the numerator of (12) and (13). Since $v'' < 0$, they are both negative. **QED**

As Proposition 1 shows, it is not necessarily true that when spillovers in public good production are present, the efficient jurisdiction size is bigger.

The intuition may be explained as follows: recall that we assume that the marginal utility of the public good is decreasing. Suppose the spillover strength θ_p increases. The agents consume more public good from surrounding jurisdictions, but at the optimum the responsiveness of the consumer's marginal valuation of the increased availability of the public good, u', may be "big" or "small". Depending on the magnitude of the responsiveness of the marginal utility, it may be optimal to increase or decrease the production of the public good. In other words, we have an "elasticity condition" for the marginal utility. If the marginal utility decreases proportionately less than what the total availability of the public good increases, this "elasticity of the marginal utility" is less than 1. In such a case, we show that it is optimal to increase production of the public good. Conversely, if the proportional decrease of the marginal utility decreases is higher than the increase in the public good, this elasticity is greater than 1. It is efficient to decrease production of the public good y. [5]

[5] The intuition of the "elasticity condition" can be understood as follows: suppose I am an eager chocolate

Now, if it is efficient to increase y, this means that the average cost each agent pays for the public good is going to be bigger than the marginal utility of crowding. In other words, we have a violation of equality (4) where the marginal benefit has to equal the marginal cost (which here is equal to the average cost of the public good). To restore equality, we have to add people to each jurisdiction, hence increase jurisdiction size, and decrease the number of localities in the economy until equality is restored again.

If it is efficient to reduce y, the average cost of the public good is less than the marginal utility of crowding. To return to equality, we have to reduce the jurisdiction size and so increase the number of localities in the economy.

The intuition for Proposition 2 is more straightforward. Since $v'' < 0$, this reinforces the effect of an increase in the spillover in crowding. It is efficient to reduce the jurisdiction size to counterbalance the spillover. But if we reduce n, the right hand side of (4) becomes larger than the left hand side. This means that the average cost of the public good is higher than the marginal disutility of crowding. To restore equality we need to reduce public good output, which is exactly what equation (12) tells us to do in this case.

4. Nash Equilibrium

In the previous section we considered only efficient allocations of the economy with public goods and spillovers to other jurisdictions. Here we suppose that entrepreneurs charge a price P and offer a mix of the private good, the public good and a total membership of the jurisdiction. The total cost of supplying the club good is αy. There are potentially an infinite number of developers and so free entry to the market for juris-

eater, and assume that the chocolate maker increases the cocoa content of the product (this is like an increase in $(1 + \theta k)$). Does that mean that I will eat more chocolate? Not necessarily; even though I like chocolate a lot, it depends on my "elasticity of the marginal utility".

dictions will drive profits to zero in equilibrium. Each developer takes the production of the public good and the membership size of the neighboring developers as given.

The typical firm offers a combination of price (in terms of the private good), public good and membership size to potential customers. To make the analysis tractable, we focus on symmetric equilibria. Thus, we assume that neighboring jurisdictions of any given locality will have the same level of public good and same population as one another. This allows us to write the entrepreneur's problem as:

$$\max_{P,y,n} \quad nP - \alpha y \tag{14}$$

$$\text{s.t.} \quad w - P + u(y + \theta_p k\bar{y}) + v(n + \theta_c k\bar{n}) \geq \bar{U} \tag{15}$$

where $\bar{U}$ is the reservation utility that agents receive in competing jurisdictions. The first order conditions for this problem are:

$$n - \lambda = 0 \tag{16}$$

$$-\alpha + \lambda u' = 0 \tag{17}$$

$$P + v' = 0 \tag{18}$$

Note that the utility constraint is binding (since the Lagrange multiplier $\lambda > 0$). This is intuitive; in equilibrium, the developer will not allow the agents to enjoy more utility than what they can expect elsewhere in the market (i.e., the developer will not give away free utility). Rearranging these equations we obtain:

$$nu' = \alpha \tag{19}$$

$$-nv' = P \tag{20}$$

Equation (19) is similar to the Samuelson condition for optimality. Equation (20) gives the optimal jurisdiction size; the total marginal disutility of one more member equals the price the entrepreneur charges for the use of the jurisdiction. These equations, together with the zero profit condition $nP = \alpha y$, (which is a consequence of free entry), gives us the solution to the firm's problem. Call this solution (y^*, n^*, P^*).

Consider now the following alternative problem: suppose that the entrepreneur maximizes the utility of each agent, subject to the agent's budget constraint:

$$\max_{x,y,n} x + u(y + \theta_p k\bar{y}) + v(n + \theta_c k\bar{n}) \text{ s.t.} \tag{21}$$

$$x + \frac{\alpha y}{n} = w.$$

Call the solution to this problem (x^{**}, y^{**}, n^{**}) and the utility associated with this solution U^{**}. It is not difficult to see that both problems are equivalent.[6] Thus, in what follows, we will be using the solution to (21) in the analysis that follows. Substituting for the expression of x of the budget constraint into the objective function, we obtain the following first order conditions:

$$nu' = \alpha \tag{22}$$

$$-nv' = \frac{\alpha y}{n} \tag{23}$$

Since we are assuming strict concavity and the Hessian to be negative definite, the solution is a maximum and is unique. Recall that the first order conditions for Pareto efficiency of section 3 were:

$$n(1 + \theta_p k)u' = \alpha \tag{3}$$

$$-n(1 + \theta_c k)v' = \frac{\alpha y}{n} \tag{4}$$

Their solution was:

$$(\bar{y}, \bar{n})_{SP}$$

Next we turn the question of how the efficient allocation $(\bar{y}, \bar{n})_{SP}$ compared to the Nash allocation (y^*, n^*).

Proposition 3. *If the spillovers of the public good are positive, the efficient production of the public good and the efficient jurisdiction size are higher than in the Nash case. Conversely, if the spillovers in public good consumption are negative, the efficient*

[6] Interested readers my apply to the authors for a formal proof.

production of the public good and the efficient jurisdiction size are smaller than the corresponding Nash magnitudes.

Proof/

Looking carefully at the Nash equilibrium and Pareto optimality first order conditions (equalities (22) and (23), and (3) and (4) respectively), we notice that the difference lies in the terms $(1+\theta_p k)$ and $(1+\theta_c k)$. Hence, we can summarize these conditions as follows:

$$-\frac{\alpha}{n} + \delta_1 u' = 0 \tag{24}$$

$$\frac{\alpha y}{n^2} + \delta_2 v' = 0 \tag{25}$$

where $\delta_1, \delta_2 = 1$ is the Nash case and $\delta_1, \delta_2 \neq 1$ is the efficient case. Our strategy of proof will be to perform comparative statics on the δ's. By this we mean moving the δs from 1 to a number different from 1 as determined by the sign of the spillovers. In effect, this moves us from the Nash equilibrium to an efficient allocation.

To show the current proposition, we keep δ_2 fixed and move δ_1. Totally differentiating the equalities above, we get:

$$\begin{bmatrix} \delta_1(1+\theta_p k)u'' & \frac{\alpha}{n^{*2}} \\ \frac{\alpha}{n^{*2}} & -\left(\frac{2\alpha y^*}{n^{*3}}\right) + \delta_2(1+\theta_c k)v'' \end{bmatrix} \begin{bmatrix} \frac{dy^*}{d\delta_1} \\ \frac{dn^*}{d\delta_1} \end{bmatrix} = \begin{bmatrix} -u' \\ 0 \end{bmatrix} \tag{26}$$

from which we obtain:

$$\frac{dy^*}{d\delta_1} = \frac{-u'\left(-\left(\frac{2\alpha y^*}{n^{*3}}\right) + \delta_2 v''\right)}{|H|} > 0 \tag{27}$$

This expression is positive, since the term in parenthesis in the numerator is negative; this we know from the second order condition. Similarly for the jurisdiction size we have:

$$\frac{dn^*}{d\delta_1} = \frac{u'\left(\frac{\alpha}{n^{*2}}\right)}{|H|} > 0 \tag{28}$$

QED

Proposition 4. *If the spillovers in crowding are positive (recall that by the nomenclature chosen, this is $\theta_c < 0$), then the optimal output of the public good and the optimal* *jurisdiction size are higher than the corresponding Nash magnitudes. If the spillovers in crowding are negative* ($\theta_c > 0$), *then the optimal output of the public good and the optimal jurisdiction size are smaller than the corresponding Nash magnitudes.*

Proof/

Again, consider equations (24) and (25). To show this proposition we move δ_2 and keep δ_1 fixed. This yields the system of equations:

$$\begin{bmatrix} \delta_1(1+\theta_p k)u'' & \frac{\alpha}{n^{*2}} \\ \frac{\alpha}{n^{*2}} & -\left(\frac{2\alpha y^*}{n^{*3}}\right) + \delta_2(1+\theta_c k)v'' \end{bmatrix} \begin{bmatrix} \frac{dy^*}{d\delta_2} \\ \frac{dn^*}{d\delta_2} \end{bmatrix} = \begin{bmatrix} 0 \\ -v' \end{bmatrix} \tag{27}$$

from which we obtain:

$$\frac{dy^*}{d\delta_2} = \frac{\frac{\alpha}{n^*}v'}{|H|} < 0 \tag{28}$$

and

$$\frac{dn^*}{d\delta_2} = \frac{-\delta_1 v' u''(1+\theta_p k)}{|H|} < 0 \tag{29}$$

QED

Hence, in the case of spillovers in the public good, y and n change in the same direction as δ. If δ_1 changes from $\delta = 1$ to $\delta > 1$ (a positive spillover), the efficient y and n are bigger than the Nash equilibrium values. In the case of negative spillovers, $\delta_1 < 1$ is the efficient case. This means that the efficient output of y and jurisdiction size n are smaller than the corresponding Nash equilibrium values.

In the case of spillovers in crowding, as δ_2 changes from $\delta_2 = 1$ to $\delta_2 > 1$, the public good and optimal membership size of the jurisdiction change inversely. The efficient magnitudes are smaller than the ones in the Nash equilibrium case.

In practice, when the spillover in the public good is positive and in crowding is negative, the two effects counterbalance each other, so that the net result is ambiguous. The efficient production of the public good and jurisdiction size are higher or lower than the corresponding Nash magnitudes.

5. Taxes and subsidies

Consider the Nash equilibrium case in the presence of spillovers only in the public good, and none in crowding ($\theta_c = 0$). Suppose that a higher authority, say the federal government, wants to achieve efficiency in the production of public goods. Therefore it decides to subsidize consumers so that they spend more on the public good y. The subsidy is assumed to be financed by a lump-sum tax t on the same consumers. How large has to be the subsidy (we assume the subsidy is a "per unit of public good" subsidy)? With the subsidy present, the developers solve:

$$\max_{x,y,n} x + u(y + \theta_p k\bar{y}) + v(n) \text{ s.t. } x + \frac{\alpha y}{n} + t = w + sy \tag{36}$$

From here we get:

$$nu' + ns = \alpha \tag{37}$$

$$-nv' = \frac{\alpha y}{n} \tag{38}$$

Comparing equations (29) and (3) we see immediately that the subsidy s has to equal $\theta_p k u'$ to achieve efficiency. The subsidy equals exactly the additional utility the agents get when consuming more of the public good y, weighted by the share in the spillover and the number of neighboring developers. Therefore we have the following proposition:

Proposition 5. *Assume the federal government subsidizes to the degree that efficient consumption of the public good y obtains with a per-unit subsidy s financed with a lump-sum tax t. Then $s = \theta_p k u'$.*

Proof/

Above.

6. Conclusion

In this paper we analyze changes in the strength of spillovers of public goods and spillovers in crowding from one locality to another. We show that in the presence of these positive or negative externalities in *public good* production and the optimal jurisdiction size may increase or decrease. However, as the externalities in *crowding* become more positive optimal jurisdiction size unambiguously increases. We also compared the socially efficient to the Nash equilibrium allocations. We find that in the presence of spillovers in the public good only or crowding only, the efficient bundle of y and n is bigger or smaller than the corresponding Nash, depending upon whether the spillovers are, respectively, positive or negative. On the other hand, when the externalities are in the public good and crowding at the same time (the case of a park, for example), it is not possible to say whether the efficient bundle is higher or smaller than the Nash allocation.

Clearly spillovers are an important real world phenomenon. The asymmetry of our results may be somewhat surprising. While it seems that spillovers in public goods consumption can have any effect on the optimum, it is unambiguous that negative spillovers in crowding imply smaller cities are efficient, while positive spillovers imply for more agglomeration is optimal. On the other hand, it comes as no surprise is that competitive forces will fail to efficiently handle these externalities. However, these results provide a foundation determining when and how higher level governments should to intervene to address such phenomenon as flight to the suburbs and urban sprawl.

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