Law of the Demand in Tiebout Economies†

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Abstract

We consider a general equilibrium local public goods economy in which agents have two distinguishing characteristics. The first is crowding type, which is publicly observable and provides external costs or benefits to the coalition the agent joins. The second is taste type, which is not publicly observable, has not external effect, and is defined over private good, public goods and the crowding profile of the jurisdiction the agent joins. The law of demand suggests that as the quality of a given crowding type (plumbers, Lawyers, Smart people, Tall people, nonsmokers, for example) increases, the compensation agents of that type receive should go down. Indeed this seems to be true on average. We provide counterexamples, however, that show that some agents of a given crowding type might actually benefit when the proportion of agents with their characteristic increases in the society. This reversal of the law of demand seems to have to do with an interaction effect between tastes and skills, something difficult to study without making these classes of characteristics distinct. We show hat this effect seems to relate to the degree of difference between various patterns of tastes. In particular, we show that there is a bound on the magnitude of this reversal that depends of the degree of continuity in the distribution of tastes of agents in the economy.

"The whole of the advantages and disadvantages of the different employments of labor and stock must, in the same neighborhood, be either perfectly equal or continually tending to equality. If in the same neighborhood there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in one case, and so many would desert it in the other, that its advantages would soon return to the level of other employments.[1]"

1. Introduction

As Adam Smith recognized, wage differentials are required to equalize the total monetary and non-monetary advantages and disadvantages amongst alternative employments; a job with favorable conditions can attract labor at relatively low wages while a job to be done under unfavorable conditions must offer a compensating wage premium if it is to attract workers. This well known, theory of equalizing differences, is suggested to be 'the fundamental market equilibrium construct in labor economics [2] and is an example of the central question we will consider in this paper.

The value of a worker's skills are determined by the how the market values the product he is able to generate. How conditions of employment are valued, however, depends on the tastes of individual worker. There is no intrinsic reason that indoor work should be preferred on the average to outdoor work, it just turns out that more workers happen to have a preferences the lean this way. Thus, when we allow for equalizing differences, the tastes of workers become important determinants of labor market equilibrium. We find that getting the most out of an economy's resources requires matching the appropriate type of worker with the appropriate type of firm:

[1] Adam Smith, *The Wealth of Nations.*

[2] S. Rosen (1986)

"the labor market must solve a type of marriage problem of slotting workers into their proper 'niche' within and between firms."[3]

It is difficult to address this process of selection process in a general equilibrium model since each commodity, including labor is treated as a homogeneous good which be allocated to productive uses with out reference to the agent who supplied it. In other words, there is a structural de-bundling of the tastes and skills of workers inherent in the model. Under these circumstances, and give diminishing marginal productively of labor, one expects a "law of demand" to hold. That is, as the quantity supplied of a given skill increases the price it receives in equilibrium should go down.

It turns out that this is really an example of a broader class of economic problems. Firms can be seen as coalitions of agents brought together in exchange for compensation to jointly produce a product. Such coalitions also form in a wide variety of other contexts including clubs, schools, groups of friends, sets of coauthors, marriages, and of course cities, towns and other jurisdictions. The question we will address in this paper is when will a law of demand hold for skills in coalition formation games. For example, will the compensation that gregarious people get from joining social groups decrease if more people become outgoing, will smart college applicants get less college aid if the population at large gets smarter, will the wage that teachers get go down if more teachers are trained, and so on.

The purpose of this paper is to explore the presence of a law of demand the context of the *crowding types model* introduced in Conley and Wooders (1996, 1997). The utility of this model in approaching this issue is that it sets up a formal distinction between the tastes and crowding effects of agents. Crowding characteristics are publicly observable and generate eternal effects on other agents. For example, they include gender, smoking preference, skills and abilities, personality characteristics, appearance, and languages spoken. Note that some of these are exogenously attached to agents (gender) and some are endogenously chosen in response to market and other incentives

[3] Rosen (1986)

(skills and professional qualifications). See Conley and Wooders(2002) for more discussion of the latter. Tastes on the other hand, are assumed to be private information and in themselves produce no external effects.

The key thing about the crowding types approach is that an agent is a bundle of tastes and skills. These things can not be taken as independent. Thus, the it is the joint distribution of these pairs and not the separate distributions of tastes and skills that will effect the equilibrium outcome of the economy. This allows us to explore explicity how the tastes of agents determine the compensating differentials needed to get agents to joint different firms/coalitions and in turn to see when a law of demand for skills will and will not hold in a Tiebout economy.

To do so we consider an coalitional economy in which small groups are strictly effective. In formally, this means that all per capita gains can be realized in groups that are small relative the size of the population and that no particular type is scarce (and thus might have monopoly power). In these circumstances we can show that the core has the equal treatment property, that is, all agents of a given type must receive the same utility in any core allocation.

Our formal question is to consider two economies that differ only in that the number of one particular crowding types is larger in one than the other. We show that at a core allocation, a law of demand holds on the average. That is the average compensation the agents possessing the crowding type that has increased in the population must go down. However, we also produce a pair of examples that shows that some agents of this relatively more abundant crowding type might actually benefit. In other words, if there are more plumbers in the world, the average plumber will be worse off. However it might be that plumbers who have a taste for working hot steam tunnels actually benefit from the overall increase in plumbers. Similarly, while computer programmers in general might oppose the free immigration of programmers from India, it might still be the case that some types of programmers (say game writer) actually benefit from the this migration.

This failure of the law of demand seems to be due to interactions between tastes and crowding characteristics, and especially, how they are bundled. To further explore this intuition, we develop an economy in which agents always have close neighbors in the taste space. That is, where tastes are epsilon close in the sense that the utility difference neighboring agents get from a given bundle is bounded by epsilon. We find in this case that this same Epsilon is bounds the degree of the reversal of the law of demand. Thus, if agents are fill the space of possible tastes densely enough, no agent of a given crowding type should benefit when the relative population of this type increases in economy.

2. The Model

We consider economies in which players are described by two characteristics, their taste types and crowding types. An agent has one of T different taste types, denoted by $t \in 1, \ldots\ldots, T \equiv \mathcal{T}$ and one of C different crowding types, denoted $c \in 1, \ldots, C \equiv \mathcal{C}$. We assume no correlation between c and t.

The total population of agents in an economy is described by a vector $N = (N_{11}, \ldots, N_{ct,} \ldots, N_{CT})$, where N_{ct} is interpreted as the total number of agents with crowding type c and taste type t. A *coalition* $m = (m_{11}, \ldots, m_{ct}, \ldots, m_{CT}) \leq N$ describes a group of agents, where m_{ct} denotes the number of agents with crowding type c and taste type t in the group. When it will not cause any confusion, we shall refer to a coalition described by m as the coalition m and the economy described by N as the economy N. The*crowding profile* of a coalition or economy m is a vector $\overline{m} = (m_1, ..., m_C)$, where $m_c = \sum_t m_{ct}$. The crowding profile simply lists the numbers of agents of each crowding type in the coalition or economy. The set of all feasible coalitions is denoted by $\mathcal{N}$. The total population in an economy or jurisdiction N is denoted $|N| = \sum N_{ct}$.

A *partition* n of the population is a set of coalitions $\{n_1, ..., n_K\}$ such that $\sum_k n_k = N$. We will write $n_k \in n$ when a coalition n_k belongs to the partition n. It will sometimes be useful to refer to individual agents whom we denote by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = \sum_{c,t} N_{ct}$. We let $\theta : \mathcal{I} \to \mathcal{C} \times \mathcal{T}$ be a mapping describing the crowding and taste types of individual agents; thus, $|\{i \in \mathcal{I}, i \in N : \theta(i) = (c,t)\}| = N_{ct}$. We will say an agent i has *type* (c,t) if $\theta(i) = (c,t)$.

With a slight abuse of notation, if individual i is a member of the coalition described by m, we shall write $i \in m$, and if i belongs to the economy described by N we write $i \in N$.

An economy has one private good x and club goods $y_1, y_2, ..., y_A$ that are provided by coalitions. The vector $y = (y_1, y_2, ..., y_A) \in \Re^A_+$ gives club good production. Each agent belongs to exactly one coalition. Each agent $i \in \mathcal{I}$ of taste type t is endowed with $\omega_t \in \Re_+$ of the private good, and has a quasi linear utility function $u_t(x, y, m) = x + h_t(y, m)$ where $i \in m$ and y is the club good production of coalition m. The cost in terms of the private good of producing y club good in coalition with membership m is given by the production function $f(y, m)$.

A particular combination of preferences and endowments for players in the economy N and production possibilities available to subsets of N is referred to as the *structure* of the economy.

We shall assume preferences satisfy *taste anonymity in consumption*(TAC), and production functions satisfy *taste anonymity in production* (TAP) defined as follows:

TAC: for all $m, \widehat{m} \in \mathcal{N}$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all $x \in \Re_+$, all $y \in \Re^A_+$, and all $t \in \mathcal{T}$ it holds that $(x, y, m) \sim_t (x, y, \widehat{m})$.

TAP: for all $m, \widehat{m} \in \mathcal{N}$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all $y \in \Re^A_+$ it holds that $f(y, m) = f(y, \widehat{m})$.

TAC and TAP capture the idea that agents care only about the crowding types and not the taste types of the agents that are in their coalition. They can be seen as defining crowding types rather than imposing restrictions on preferences. To illustrate, the cost of production depends on the skill mix of the people in the jurisdiction, but whether or not skilled workers like warm or cool climates is of no relevance. As for consumption, we might care about the age of other people but are indifferent to whether or not they are danger averse.[4] We will assume throughout that all economic structures satisfy both TAC and TAP.

A *feasible state* of the economy $(X, Y, n) \equiv ((x_1, \ldots, x_I), (y_1, \ldots y_K), (n_1, \ldots n_K))$ consists of a partition n of the population, an allocation of private goods to agents $X = (x_1, \ldots, x_I)$ and a club goods production plan for each coalition, $Y = (y_1, \ldots y_K)$ such that

$$\sum_k \sum_{ct} n^k_{ct} \omega_t - \sum_i x_i - \sum_k f(y^k, n^k) \geq 0.$$

We also say that (x, y) is a *feasible allocation for a coalition* m if

$$\sum_{c,t} m_{ct} \omega_t - \sum_{i \in m} \overline{x}_i - f(\overline{y}, m) \geq 0$$

A coalition $m \in \mathcal{N}$ producing a feasible allocation $(\overline{x}, \overline{y})$ *can improve upon* a feasible state (X, Y, n) if for all $i \in m$,

$$u_t(\overline{x}_i, \overline{y}, m) > u_t(x_i, y_k, n_k).$$

where in the original state $i \in n_k$ and $n_k \in n$. A feasible state of the economy (X, Y, n) is a *core state of the economy* or simply a *core state* if it cannot be improved upon by any coalition m. This simply says that a feasible state is in the core if it is not possible for a coalition of agents to break away and, using only their own resources, provide all its members with preferred consumption bundles.

This paper will focus solely on economies in which small groups are effective. An economy satisfies *strict small group effectiveness*, SSGE, if there exists a positive integer B such that:

1. For all core states (X, Y, n) and all $n_k \in n$, it holds that $|n_k| < B$
2. For all $c \in \mathcal{C}$ and all $t \in \mathcal{T}$ it holds that $N_{ct} > B$./

[4] You may well indirectly care about the tastes of agents you live with through the coalitions eventual choice of y. However, given y, TAC and TAP imply your welfare does not directly depend on the tastes of agents.

SSGE is a relatively strong formalized version of the sixth assumption in Tiebout's paper that there be "an optimal community size" - condition one stating that any coalition with more than B agents can be improved upon while condition two says that this limit of B is small relative to a population which contains at least B agents of each type. As recent literature shows, however, economies satisfying apparently mild conditions can be approximated by ones satisfying SSGE (cf., Kovalenkov and Wooders 1999 and references therein).

2.1 Equal Treatment

The first result follows immediately from SSGE and shows that any core state must have the *equal treatment property*, that is any two agents of the same type must be equally well off in any core state.

Theorem 1. *Let (X, Y, n) be a core state of an economy satisfying SSGE. For any two individuals $i, \hat{\imath} \in \mathcal{I}$ such that $\theta(i) = \theta(\hat{\imath}) = (c, t)$, if $i \in n^k$ and $\hat{\imath} \in n^{\hat{k}}$ then $u_t(x_i, y, n^k) = u_t(\hat{x}_{\hat{\imath}}, \hat{y}, n^{\hat{k}})$.*

Proof/

See Conley and Wooders (1997)

One consequence of this result is that for any core state (X, Y, n) we can associate a vector of payoffs $u = (u_{11}, ..., u_{ct}, ..., u_{CT}) \in \Re^{CT}$ where u_{ct} is the utility of an agent with crowding type c and taste type t.

Note that Theorem 1 can not be directly verified by looking at observable data. Wages received by agents of a given type could be wildly different provided the nonobservable nonmonetry compensations of joining a coalition offset these. The next result provides a directly observable counter part to this.

Theorem 2. *Let (X, Y, n) be a core state of an economy satisfying SSGE. Suppose that for some jurisdiction $n^k \in n$, for some crowding type $c \in \mathcal{C}$, and for two taste types $t, t' \in \mathcal{T}$, $n^k_{ct} > 0$, $n^k_{ct'} > 0$. Then for all $i, j \in k$ such that $\theta(i) = (c, t)$ and*

$\theta(j) = (c, \widehat{t})$, *it holds that,*

$$\omega_t - x_i = \omega_{\widehat{t}} - x_{\widehat{i}} \equiv \rho_c(y^k, n^k)$$

Proof/

See Conley and Wooders (1997)

Theorem 2 says that the side payment (which might be positive or negative) that an agent receives/offers to join a jurisdiction depend only on the agents *crowding type.* Thus, these side payments are a kind of anonymous price that depends only on the observable and externally relevant characteristics of an agent and not his unobservable tastes. Note the contrast between these prices and Lindahl prices in this respect.

From now on we will use $\rho_c(y, m)$ to denote the *admission price for players of crowding type c to enter the coalition m producing y of the club good.* For the special case of firm formation, these admission prices will be generally be negative and are interpreted as the wages paid by firms to workers.

2.2 Core equivalence

Elaborating on the above *Tiebout price system for crowding type c* associates to each possible club good level and possible coalition (containing at least one player with crowding type c) an admission price, which applies to all players of crowding type c. Thus players know the price to join any possible jurisdiction and we also see that prices are anonymous in the sense that they do not depend on the tastes of agents.[5] A *Tiebout price system* is simply a collection of price systems, one for each type, and is denoted by ρ.

We define a *Tiebout equilibrium* as a feasible state $(X, Y, n) \in F$ and a Tiebout price system ρ such that

[5] Formally we also require that for all $m, \widehat{m} \in N$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all y it holds that $\rho(y, m) = \rho(y, \widehat{m})$.

1. For all $n^k \in n$, all individuals $i \in n^k$ such that $\theta(i) = (c, t)$, all alternative jurisdictions $m \in \mathcal{N}_c$, and for all levels of public good production $y \in \Re^A_+$,

$$\omega_t - \rho_c(y^k, n^k) + h_t(y^k, n^k) \geq \omega_t - \rho_c(y, m) + h_t(y, m)$$

2. For all potential jurisdictions $m \in \mathcal{N}$ and all $y \in \Re^A_+$,

$$\sum_{c,t} m_{ct}\rho_c(y, m) - f(y, m) \leq 0$$

3. For all $n^k \in n$,

$$\sum_{c,t} n^k_{ct}\rho_c(y^k, n^k) - f(y^k, n^k) = 0$$

Thus a Tiebout equilibrium is a decentralized market equilibrium. Condition 1 states that, given the prices they face to join coalitions, every player is in his preferred jurisdiction. Condition 2 states that, given the price system, no new coalition could make positive profits while existing coalitions make zero profit.[6]

Under strict small group effectiveness, a strong result can be proven about the relationship between the core and Tiebout Equilibrium:

Theorem 3. *If an economy satisfies SSGE then the set of states in the core of the economy is equivalent to the set of Tiebout equilibrium states.*

Proof/

See Conley and Wooders (1997)

Theorem 3 confirms that in the crowding types model efficient allocations can be decentralized through an anonymous price system. Thus, when we consider firm formation, all workers can choose amongst jobs to maximize their utility and the resulting outcome will be an efficient stable outcome in which the right types of workers are matched to the right type of firms.

Thus, the crowding types model allows us to model firm, jurisdiction or region formation, taking account of both the tastes of workers and their productivity. As such,

[6] From a firm perspective this does not imply the firm makes zero profit, it means that any profit has been redistributed to the workers and owners of that firm.

it gives us a reasonably complete way to model the theory of equalizing differences. The rest of the paper will reflect this by applying the model to consider the relevance of the law of supply when equalizing differences are present in the labor market.

3. The Law of Demand

In this section, we formally develop positive and negative results regarding the law a demand. This is done by way a comparative static exercise in which two economies. These economies have identical technology, and identical population of all agents except for one particular crowding type, c. For this one type c the second economy has an increased population spread in some arbitrary way across taste types. Thus, crowding types the two economies have the same number of plumbers who like football, plumbers who like hockey, plumbers who like baseball, lawyers how like football, lawyers who like hockey, lawyers who like baseball etc. However, the second economy might have twice as many doctors who like football, one additional doctor who likes hockey and the same number of doctors who like baseball.

More formally, consider two economies S and G with player sets $S = (S_{11}, \ldots, S_{ct,} \ldots, S_{CT})$ and $G = (G_{11}, \ldots, G_{ct,} \ldots, G_{CT})$, where S_{ct} is interpreted as the total number of agents with type (c, t) in economy S and where G_{ct} is interpreted as the total number of agents with type (c, t) in economy G. If $u^s = (u^s_{11}, ..., u^s_{ct}, ..., u^s_{CT}) \in \Re^{CT}$ and $u^g = (u^g_{11}, ..., u^g_{ct}, ..., u^g_{CT}) \in \Re^{CT}$ represent core payoffs in the equal treatment core of economies S and G respectively then it can be shown (Kovalenkov and Wooders 2002) that

$$(u^s - u^g) \cdot (S - G) \leq 0$$

One immediate consequence of this is that a ceteris paribus increase in the number of players with a particular type (that is, a particular $\{c, t\}$ combination cannot be beneficial to players of that type.

More formally, $u^s_{ct} \geq u^g_{ct}$ if $S_{ct} < G_{ct}$ and $S_{c't'} = G_{c't'}$ for all other c' and t'. Thus, a law of demand applies on a type by type basis. The problem with this is that taste types are not observable. Thus, the data will not tell us anything of the relative increases of a given type.

Of more interest is a ceteris paribus increase in the number of players with a particular crowding type. The following result shows that not all players of a crowding type can gain if there is a ceteris paribus increase in the number of players of that crowding type and on average, must lose.

Proposition 1. *If $S_{ct'} \leq G_{ct'}$ for all $t' \in \mathcal{T}$ and $S_{c't'} = G_{c't'}$ for all $c' \in \mathcal{C}, c' \neq c$ and all $t' \in \mathcal{T}$ then $u^s_{ct} \geq u^g_{ct}$ for at least one type t and, moreover, if $u^s_{ct'} < u^g_{ct'}$ for some type t' then there exists some t such that $u^s_{ct} > u^g_{ct}$.*

Proof/

There are two cases: (1) $u^s_{ct} = u^g_{ct}$ for all $t \in \mathcal{T}$ in which case the Corollary is trivial. (2) There exists some $t' \in \mathcal{T}$ such that $u^s_{ct'} < u^g_{ct'}$. Given that $(u^s_{c1} - u^g_{c1})(S_{c1} - G_{c1}) + (u^s_{c2} - u^g_{c2})(S_{c2} - G_{c2}) + ... + (u^s_{cT} - u^g_{cT})(S_{cT} - G_{cT}) \leq 0$ and $(u^s_{ct'} - u^g_{ct'})(S_{ct'} - G_{ct'}) > 0$ there must exist some t such that $(u^s_{ct} - u^g_{ct})(S_{ct} - G_{ct}) < 0$.■

4. Failures of the Law of Demand

Proposition 1 shows that on the average a law of demand hold for crowding types. Thus, Lawyers as whole lose when more lawyers join the bar. (Of course economists gain when more economists join the bar, but this is different kind of crowding effect.) In this section we provide two examples that show the counter-intuitive result that law of demand need not hold for all agents when the crowding type they posses increases. The first example treats crowding in consumption and the second crowding in production.

Example 1: There are 3 taste types - people who like music at work (L), hate music at work (H) and do not mind some music at work (I). There are 3 crowding types - people who sing/whistle at work (W), do not sing (D) and occasionally sing. (O). People join together to form partnerships and produce a good, say a building service. Note that all agents are equally productive in production of the good. An agent's utility from a partnership depends on his tastes and the crowding profile of the partnership.

$U_H(W,W) = 0$ $U_I(W,W) = 2$ $U_L(W,W) = 4$
$U_H(W,O) = 1$ $U_I(W,O) = 2$ $U_L(W,O) = 3$
$U_H(W,D) = 2$ $U_I(W,D) = 2$ $U_L(W,D) = 2$
$U_H(O,O) = 2$ $U_I(O,O) = 2$ $U_L(O,O) = 2$
$U_H(O,D) = 3$ $U_I(D,D) = 2$ $U_L(O,D) = 1$
$U_H(D,D) = 4$ $U_I(O,D) = 2$ $U_L(D,D) = 0$

For example, if someone who sings at work but does not like music at work joins with someone who does not sing at work he receives payoff $U_H(W,D) = 2$. The value function is as follows:

composition	total utility	composition	total utility	composition	total utility
WL, WL	8	OL, OL	4	DL, DL	0
WI, WI	4	OI, OI	4	DI, DI	4
WH, WH	0	OH, OH	4	DH, DH	8
WH, WL	4	OH, OL	4	DH, DL	4
WH, WI	2	OH, OI	4	DH, DI	6
WI, WL	6	OI, OL	4	DI, DL	2
WL, OL	6	DL, OL	2	DL, WL	4
WI, OI	4	DI, OI	4	DI, WI	4
WH, OH	2	DH, OH	6	DH, WH	4
WH, OL	4	DH, OL	4	DH, WL	4
WL, OH	4	DL, OH	4	DL, WH	4
WH, OI	3	DH, OI	5	DH, WI	4
WI, OH	3	DI, OH	5	DI, WH	4
WI, OL	5	DI, OL	3	DI, WL	4

WL,	OI	5	DL,	OI	3	DL,	WI	4

We contrast two economies where the number of players of each type is:

type	WH	WI	WL	OH	OI	OL	DH	DI	DL
number of type in economy S	6	4	4	2	2	4	4	4	4
number of type in economy G	6	4	4	4	4	6	4	4	4

Note that the number of players with crowding type O has increased. Furthermore the number of players with types OH, OI and OL has increased by the same number, namely 2.

Two possible core allocations can be detailed as follows:

1. Economy S: 2x(DH, DH), 4x(WI, OL), 2x(WL, WL), 2x(OH, DI), 4x(WH, DL), 2x(WH, DI) and 1x(OI, OI).
2. Economy G: 2x(DH, DH), 2x(WL, WL), 2x(WH, OL), 4x(WI, OL), 4x(OH, DI), 4x(WH, DL) and 2x(OI, OI)

Giving core payoffs:

type	WH	WI	WL	OH	OI	OL	DH	DI	DL
payoff in economy S	2	3	4	3	2	2	4	2	2
payoff in economy G	1.5	2.5	4	2	2	2.5	4	3	2.5

We observe that agents of type OL receive a higher payoff in economy G despite the increase in agents with crowding type O and type OL. So why are agents of type OL able to gain? Given that agents of type OL like to listen to music they would naturally want to form a partnership with agents who whistle (crowding type W) as opposed to those who do not whistle (D). Conversely, agents of type OH would naturally want to form a partnership with agents who do not whistle (D) as opposed to those who do (W). In economy S it so happens that agents with crowding type W are doing relatively well and agents with crowding type D relatively poorly; this has the knock on effect that agents of type OL receive a relatively low payoff and agents of type

OH a relatively high payoff. In economy G the increased number of agents of type OH sees their 'bargaining position' reduced and consequently their payoffs fall. This feeds through into an increased 'bargaining power' for those agents who do not whistle and a decreased bargaining power for those who whistle. As the 'bargaining power' of whistlers falls agents of type OL are able to increase their payoff. Basically, there are cross type influences whereby agents of type OL 'gain more bargaining power' by the increased number of players of type OH than they lose by the increased number of players with their own type OL.

Example 2: There are three taste types - those who like working outdoors (O), indoors (D) or both (B). There are three crowding types - plumbers (P), gardeners (G) and laborers (L). Agents form partnerships and can choose to offer a gardening service, general laboring service or plumbing service. The profits a partnership can make depends on the crowding composition of the partnership and their choice of service to provide:

Composition	garden	labor	plumbing
GG	20	10	5
GL	19	20	15
GP	15	20	15
LL	15	20	15
LP	19	20	18
PP	5	10	20

For example, a gardener and a laborer can make profit of 20 from setting up a general laboring service. The utility an agent receives depends on the type of service the partnership is providing:

taste type	garden	labor	plumbing
O	5	4	1
B	3	5	4

I	0	2	4

Agents only care who they form a partnership with in that it effects the profits of the partnership. The value to all possible jurisdictions is given as follows:

composition	total utility	composition	total utility	composition	total utility
GO, GO	30	PO, PO	22	LO, LO	28
GB, GB	26	PB, PB	28	LB, LB	30
GI, GI	20	PI, PI	28	LI, LI	24
GO, GI	25	PO, PI	25	LO, LI	26
GB, GI	23	PB, PI	28	LB, LI	27
GO, GB	28	PO, PB	25	LO, LB	29
GO, PO	28	LO, PO	29	GO, LO	29
GB, PB	30	LB, PB	30	GB, LB	30
GI, PI	24	LI, PI	26	GI, LI	24
GO, PI	26	LO, PI	26	GO, LI	26
GI, PO	26	LI, PO	26	GI, LO	26
GB, PI	27	LB, PI	27	GB, LI	27
GI, PB	27	LI, PB	27	GI, LB	27
GO, PB	29	LO, PB	29	GO, LB	29
GB, PO	29	LB, PO	29	GB, LO	29

We contrast two economies where the number of players of each type is:

type	GO	GB	GI	PO	PB	PI	LO	LB	LI
number of type in economy S	2	6	4	12	2	2	6	2	2
number of type in economy G	2	6	4	12	2	2	12	4	4

Note that the number of agents with crowding type L has increased. Further, it is an 'equi-proportional' increase in that the number of agents with types LO, LB and LI doubles. Two possible core outcomes can be detailed:

1. Economy S: 6x(PO, LO), 4x(PO, GB), 2x(PO, GI), 1x(LI, LI), 2x(LB, GI), 2x(GB, PB), 1x(GO, GO), 1x(PI, PI)
2. Economy G: 12x(PO, LO), 2x(GB, PB), 4x(LB, GI), 4x(LI, GB), 1x(GO, GO), 1x(PI,PI).

type	GO	GB	GI	PO	PB	PI	LO	LB	LI
payoff in economy S	15	15	12	14	15	14	15	15	12
payoff in economy G	15	14.5	11.5	14.5	15.5	14	14.5	15.5	12.5

Observe that the payoff of agents with types LB and LI increase despite the increased number of laborers. The reason for this increase in payoffs appears to come from the role of laborers who like to work outdoors (LO) and gardeners who like to work both outdoors and indoors (GB). In economy S it so happens that agents of types LO and GB are receiving relatively high payoffs. As the number of agents with type LO increases their 'bargaining power' is significantly reduced. The knock on effect is that payoffs of agents with types LO and GB fall and the payoffs of agents of types PO, LB and LI increase. The 'bargaining power' of type LB and LI agents is increased more by the larger number of type LO agents than by the larger number of type LB and LI agents.

5. Law of demand result

We will say that *tastes are ε close* if

$$|h_t(y,m) - h_{t+1}(y,m)| < \varepsilon \tag{1}$$

for all $t = 1, ..., T-1$, all m and all y where $\varepsilon \geq 0$.

Lemma 1. *If tastes are ε close and u is a vector of core payoffs then*

$$|u_{ct} - u_{ct+1} - \omega_t + \omega_{t+1}| < \varepsilon \tag{2}$$

for all $t = 1, ..., T-1$ *and all* c.

Proof/

Let (X, Y, n) be a core state. Chose any $c \in \mathcal{C}$ and $t \in \mathcal{T}$ and suppose first for that

$$u_{ct} - u_{ct+1} - \omega_t + \omega_{t+1} > \varepsilon.$$

Rewritten this implies:

$$u_{ct+1} < u_{ct} - \varepsilon - \omega_t + \omega_{t+1}$$

Consider any agent $i \in \mathcal{I}$ such that $\theta(i) = (c, t)$ and let $i \in\in n_k \in n$, where the public good level is y_k and the admission price is $\omega_t - x_i = \rho_c(y_k, n_k)$. Now consider another agent $j \neq i$ where $\theta(j) = (c, t+1)$, and $j \notin n_k$ If he were to replace agent i in jurisdiction n_k (paying the same admission price) his utility would be:

$$\bar{u}_{ct+1} = h_{t+1}(y_k, n_k) + \omega_{t+1} - \omega_t + x_i.$$

Since tastes are ε close, rearranging lets us conclude

$$h_t(y_k, n_k) - \varepsilon \leq h_{t+1}(y_k, n_k) \leq h_t(y_k, n_k) + \varepsilon$$

Substituting we find:

$$\bar{u}_{ct+1} \geq h_t(y_k, n_k) - \varepsilon + \omega_{t+1} - \omega_t + x_i.$$

or

$$\bar{u}_{ct+1} \geq u_{ct} - \varepsilon + \omega_{t+1} - \omega_t.$$

This implies that $\bar{u}_{ct+1} - u_{ct+1} > 0$. Since all agents who remained in n^k are exactly as well off by TAC, this is a blocking coalition contradicting that we started at a core allocation.

A similar contradiction can be reached if

$$u_{ct} - u_{ct+1} - \omega_t + \omega_{t+1} < -\varepsilon.$$

by doing to reverse agent substitution (that is, putting i into j's initial jurisdiction). Putting these two inequalities together we conclude that

$$|u_{ct} - u_{ct+1} - \omega_t + \omega_{t+1}| < \varepsilon$$

■

Define the following:

Monotonicity in Crowding Types with bound δ/ Consider two economies S and G and let u^s and u^g represent core payoffs in the equal treatment core of economies S and G respectively. If $u^s_{ct} \geq u^g_{ct}$ for all $t \in \mathcal{T}$ whenever $S_{ct'} \leq G_{ct'}$ for all $t' \in \mathcal{T}$ and $S_{c't'} = G_{c't'}$ for all $c' \neq c$ and all t'.

Proposition 2. *Given any real number $\delta > 0$ there exists real number $\varepsilon > 0$ such that if tastes are ε close payoffs satisfy MCT with bound δ.*

Proof/

By Corollary 1 there must be one taste type t such that $u^s_{ct} \geq u^g_{ct}$. By Lemma 1, $|u^s_{ct} - u^s_{ct'}|, |u^g_{ct} - u^g_{ct'}| < \delta/2$ if tastes are $\delta/2T$ close.■

6. Conclusion

We began this paper by introducing, and explaining the importance of, the theory of equalizing differences. This paper has provided a new theoretical approach to modeling equalizing differences by drawing on an analogy between local public goods and the non-wage attributes of jobs. That is, the attributes which necessitates equalizing differences, such as danger, cleanliness, climate and the range of local amenities can all be seen as club goods.

The analogy of local public goods led us to consider the crowding types model of Conley and Wooders. This model has many desirable properties from a public economic sense and we find these qualities equally useful in the context of firm and region formation. Thus, the model allowed us to present a complete model of equalizing differences in which we can account for the compensating wages between differing taste types while also modeling the markets for different productivity and skill levels. However, this did not come at the cost of unduly restrictive assumptions. We do assume free mobility, no redistribution between coalitions (e.g. no governments) and perfect information on the types of jobs available, but these are standard assumptions for the topic. Perhaps of more concern, we assume that a players crowding type is observable and that crowding types are independent of taste types. Both these assumptions are unrealistic, to illustrate, consider the well known problem of workers who shirk - whether or not a worker is a shirker alters his crowding type yet this is not observable. However, as a simplification both assumptions can be justified.

Another question of concern in modeling firm formation is how we can model the interaction between firms. In the local public good literature, jurisdictions are seen as self contained but this cannot be extended to a firm context as firms and the workers of firms rely on other firms in the natural exchange process of an economy. This raises complications through the cost function to produce the club good. The cost function, represents the outcome of a market equilibrium in which the demand for the product that the firm will produce (with the club good as a by-product) is determined. However, this demand will depend on the number of other firms producing this product. For example, if only one firm is producing steel then we would expect this firm earns a large revenue, so the cost for the club good produced as a by product of steel is very small. However, if there is a surplus of steel, the revenue from producing steel is low and the cost of the club good is thus relatively high. This is an example of how the cost function to one coalition depends on the actions of other players and other coalitions. This paper has not addressed that issue, assuming the costs are independent of other players actions, as in the local public good literature, however, this is clearly an issue that needs to be considered in more detail.

Having introduced the model, we turned to an application of particular interest -

the law of supply, which states that following a ceteris paribus increase in the supply of a factor of production the return to that factor cannot increase. The introduction of compensating differentials means that taste types become important parts of the labor market - if one player prefers the attributes of the firm or region you can afford to pay that person a lower wage. This creates an independence in the money wage that players with the same skills, but different tastes, can earn and as such the arbitrage to equalize wages that we would expect within the standard market paradigm no longer apply. As such, we asked the question of whether we can guarantee the law of supply and found that in general we could not do so even when we put strict restrictions on the composition of the population change.

A natural extension would be to consider the possibility of approximate MCT. That is, we cannot guarantee MCT but can guarantee that any contradiction to MCT involves negligible payoff changes. In turn, this paper has focused on when an economic structure satisfies MCT, i.e. for all types payoffs satisfy MCT, but we may want to restrict attention to guaranteeing that the payoffs to certain types satisfy MCT. The last section on continuity is illustrative of these two extensions. We argued, informally above, for example, that assuming a normal distribution to tastes may allow us to prove that payoffs to 'average people' approximately satisfy MCT even if, for those on the extremes of the taste distribution, there are significant counter examples to MCT. Continuity is difficult to introduce into the crowding types model, however, if the distribution of tastes can be modeled effectively it allows us to begin analyzing these issues of approximate MCT.

¿From the general perspective of modeling equalizing differences there remains one significant area of further study. Compensating differentials apply to a wide variety of attributes of which the majority can be modeled as above. The model can be used to look at regional compensations because of climate, local amenities and scenery etc. We have also considered firm and individual specific attributes which can include cleanliness, vacations, shift work, pension packages, probability of unemployment and danger etc. The results above, however, do not apply to compensating differentials on the basis of human capital. That is, we have not considered the equalizing variations resulting from the cost and time spent learning a trade or skills. To do so would require us to look at the model from a different perspective - we have been comparing the payoff to players with the same crowding type but different taste type, while modeling human capital would require us to consider the payoffs to players with the same tastes but different crowding type. This paper shows the way to do this, however, the issue of human capital neatly fits the model of genetic types introduced in Conley and Wooders (2000). This paper generalizes the crowding types model so that players are endowed with a genetic type and not a crowding type. Players then purchase their crowding type with costs dependent on their genetic type. For example, the genetic type may be the level of intelligence and people purchase their skill level, with players with a higher intelligence finding it cheaper to purchase a high skill level. This question naturally fits the issue of human capital and as such would allow us to present a very interesting discussion of the role education and training plays in the process of equalizing differences.

One further issue we note for future consideration is the possibility of players belong to more than one jurisdiction. That is, a person joins a firm, then chooses the type of region he wants to live in and finally chooses the type of jurisdiction, meaning that an agent belongs to three distinct coalitions, or alternatively an agent may belong to two firms. This opens up a whole range of issues as to how the model can be extended and what we can learn from doing so.

In conclusion, this paper has presented a new way of considering two very old economic issues. Using the crowding types model we have analyzed the process of compensating differentials in the labor market and applied this to question the law of supply. The crowding types model has previously only been used to model public good economies but clearly it can have a very interesting role to play in modeling firm formation. This paper has merely looked at one possible application but there are a whole range of issues that still remain to be studied.