Finite Decentralization in a Tiebout Economy with Crowding Types†

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Abstract

We study a model with local public goods in which agents' crowding effects are formally distinguished from their taste types. It has been shown that the core of such an economy can be decentralized with anonymous admission prices (which are closely related to cost share prices). Unfortunately, such a price system allows for an arbitrary relationship between the public goods level in a given jurisdiction and the cost to an agent for joining. Formally, this means that admission prices are infinite dimensional. Attempts to decentralize the core with finite price systems such as Lindahl prices suggest that this is possible only under fairly restrictive conditions. In this paper, we introduce a new type of price system called *finite cost shares*. This system has strictly larger dimension than Lindahl prices but, in contrast to general cost share prices, is finite. We show that this allows for decentralization of the core under much more general conditions than are possible with Lindahl prices.

1. Introduction

The traditional Lindahl equilibrium concept requires that agents with different tastes face different prices for public goods. As Samuelson (1954) pointed out, this gives agents an incentive to misrepresent their preferences and makes it unlikely that a market mechanism based on such prices would be able to efficiently provide public goods. In response, Tiebout (1956) argued that many types of public goods are "local" rather than "pure" and that competition among providers of local public goods would induce effective preference revelation as consumers "vote with their feet" for the community providing public good levels which best suit their tastes.

Tiebout's argument provided the seed for a large theoretical literature that explores the possibility of decentralization of optimal local public goods provision in a variety of contexts.[1] One problem uncovered by this research is that when crowding is differentiated (that is, when the crowding effects of an agent depend upon his type), in general decentralization with anonymous prices is not possible. Conley and Wooders (1997a) observe that anonymous decentralization is not possible because the existing differentiated crowding models tied together an agent's tastes and external effects and propose a new model in which these two characteristics are formally distinguished from one another. The new "crowding types" model allows them to show that the core can be decentralized using a Tiebout admission price system that specifies a single price that an agent of a particular crowding type must pay to join a jurisdiction with a specific crowding profile and level of public goods. The important feature of the Tiebout price system is that it is anonymous in the sense that it depends only on an agent's observable crowding type and not on his unobservable tastes.

A significant criticism of the Tiebout price system, however, is that the admission prices generally are infinite dimensional— that is, one price is required for each crowding type, for each jurisdiction, and for each of the continuum of possible public good levels. One has to be a little bit suspicious about the real-world relevance of

[1] Early papers include Pauly (1970), McGuire (1974), Berglas (1976), Wooders (1978), and Boadway (1980). See Conley and Wooders (1997b) or Cornes and Sandler (1996) for recent surveys.

such prices. Clearly, they would be very costly for firms to specify and very difficult for consumers to understand. To address the concern of infinite dimensional prices, Conley and Wooders (1998) study the relationship between the core and the set of anonymous and nonanonymous Lindahl price equilibria.[2] Anonymous Lindahl prices specify a participation fee and a per unit price for public goods which do not depend on private information. In contrast, nonanonymous Lindahl prices specify a participation fee and a per unit price for public goods which may depend on private information. Under standard convexity conditions, they find that the core and set of *nonanonymous* Lindahl equilibrium states are equivalent.[3] However, except when public goods are produced under constant returns to scale and there is only one crowding type (crowding is anonymous), the core is generally larger than that of *anonymous* Lindahl equilibrium states, confirming the results of Wooders (1978) for anonymous Lindahl equilibria. Thus, except for the class of economies with linear technology and one type of agent, the existing literature leaves us with a choice of anonymous decentralization with an infinite dimensional price system or nonanonymous decentralization with a finite dimensional price system.

The pure public goods literature considers pricing systems heretofore unexplored in the context of local public goods economies. Kaneko (1977a,b) was the first to introduce the notion of cost share equilibrium to a public goods economy. He proves existence of a cost share equilibrium in an economy with several public goods and shows that the core of a specific voting game and the set of cost share equilibria coincide.[4] Mas-Colell (1980) characterizes Pareto optimal and core allocations and explores their relation to the cost sharing equilibria. Mas-Colell and Silvestre (1989) define a cost share equilibrium concept which is a generalization of the Lindahl equilibrium concept and

[2] Barham and Wooders (1998) also explore Lindahl pricing in local public goods economies.

[3] See also Wooders(1989, 1997) for related results.

[4] Kaneko actually defined the ratio equilibrium for an economy with several public goods. The subsequent literature has focused on the single public good case and has settled on the term “cost share equilibrium”. Also see Diamantaras and Wilkie (1994) who generalize his concept by allowing for multiple private goods and, more significantly, permitting public goods to be inputs.

which endogenously derives Lindahl equilibrium profit shares. Weber and Wiesmeth (1991) show equivalence between the set of Lindahl equilibria described by Mas-Colell and Silvestre and the set of linear cost share equilibria. Weber and Wiesmeth also find upper and lower bounds on the relative marginal rates of substitution which characterize the set of core allocations which can be decentralized by a linear cost share equilibrium. Also see Diamantaras and Gilles (1996) and Gilles and Diamantaras (1998) who further extend this research to economies with public projects and multiple private goods and study the relationship of various notions of the core and cost share equilibria (often finding negative results).

The purpose of this paper is to generalize the anonymous decentralization results for local public goods economies, and to connect these results to the public goods literature mentioned above. More specifically, we define a type of cost share price system for local public goods economies that is more general than Lindahl prices, but retains the feature that prices have finite dimension. We call this a *finite cost share price system.* Following Weber and Wiesmeth (1991), we define a bounding condition on marginal rates of substitution and transformation called BAARS. We show that if the economy satisfies strict small group effectiveness and BAARS, then the core and nonanonymous finite cost share equilibria are equivalent. This generalizes the class of economies for which finite decentralization is possible in that we do not require convexity either in preferences or production. We also show that with a strengthening of BAARS, but without assuming convexity, if crowding is anonymous then the core and *anonymous* finite cost share equilibrium states are equivalent. This contrasts with Wooders (1978) who assumes that production is linear and preferences convex to show decentralization with anonymous Lindahl prices.

2. A Local Public Goods Model

We consider an economy with one private good and one local public good. There are I agents indexed $i \in \{1, \ldots, I\} \equiv \mathcal{I}$. In Tiebout economies, agents typically find it optimal to self-select into many "small" jurisdictions for the purpose of consuming local public goods. An arbitrary jurisdiction of agents is denoted by $s \subset \mathcal{I}$ and $\mathcal{S}$ denotes the set of all possible jurisdictions. A list of jurisdictions $\{s^1, \ldots, s^P\} \equiv S$ is a *partition* if $\cup_p s^p = \mathcal{I}$ and $s^p \cap s^{\overline{p}} = \emptyset$ for all $s^p, s^{\overline{p}}$ such that $p \neq \overline{p}$.

Agents are distinguished by two factors: tastes and publicly observable crowding types. We allow each agent to have one of T different sorts of tastes or preferences, denoted by $t \in \{1, \ldots, T\} \equiv \mathcal{T}$. The mapping $\tau : \mathcal{I} \to \mathcal{T}$ assigns a taste type to each agent in the economy. That is, if agent i is of taste type t, then $\tau(i) = t$. Each agent also possesses one of C different sorts of publicly observable crowding types, denoted $c \in \{1, \ldots, C\} \equiv \mathcal{C}$. The mapping $\kappa : \mathcal{I} \to \mathcal{C}$ assigns a crowding type to each agent in the economy. The *crowding profile of a jurisdiction* $s \in \mathcal{S}$ is the mapping $\mathcal{K} : \mathcal{S} \to Z_+^C$ given by[5]

$$\mathcal{K}(s) \equiv \big(|s_1|, \ldots, |s_C|\big),$$

where $i \in s_c$ if and only if $i \in s$ and $\kappa(i) = c$, and where $| \bullet |$ denotes the cardinality of a set. Thus, we denote an arbitrary profile of crowding characteristics by $n = (n_1, \ldots, n_C) \in Z_+^C$ where n_c is the number of agents with crowding type c.

Finally, if agent $i \in \mathcal{I}$ has taste type t and crowding type c, we can represent that agent's overall *type* by the mapping $\theta : \mathcal{I} \to \mathcal{C} \times \mathcal{T}$. That is, if $\theta(i) = (c, t)$ then $\kappa(i) = c$ and $\tau(i) = t$. To further identify groups of individuals possessing a proscribed characteristic, we let $\mathcal{S}_c \subset \mathcal{S}$ denote the set of jurisdictions which contain at least one individual with crowding type c. We similarly define $\mathcal{S}_t$ and $\mathcal{S}_{ct}$.

In this economy, agents are members of exactly one jurisdiction and consume one private good, x, and one public good, y. Each agent i has an endowment of private good $\omega_{\tau(i)}$ and a utility function $u_{\tau(i)} : \Re \times \Re_+ \times Z_+^C \to \Re$. More specifically, if $\theta(i) = (c, t)$, then the utility function for agent i is given by $u_t(x_i, y, n) = x_i + h_t(y, n)$, where x_i is a level of private good, y is a level of public good, and n is the crowding profile of

[5] Here and throughout we use the convention that Z denotes the set of integers.

the jurisdiction in which i resides. Note that this specification of preferences ensures that agents are only directly affected by the observable crowding types of other agents and not their unobservable preferences. In addition to quasi-linearity, we assume that preferences are monotonic in the public good.

Monotonicity in Consumption: $h_t(y,n) \geq h_t(\overline{y},n)$ for all $t \in \mathcal{T}$, $n \in Z_+^C$, and $y, \overline{y} \in \Re_+$ such that $y \geq \overline{y}$.

The crowding profile of a jurisdiction also affects public good production. The production technology is given by the cost function $f : \Re_+ \times Z_+^C \to \Re$ where $f(y,n)$ is the cost in terms of private good of producing y public good in jurisdiction with crowding profile n. As with utility functions, this specification ensures that the production capabilities of a jurisdiction depend only on its crowding profile and not on consumers' preferences. We assume that the cost of producing the public good is strictly monotonic.

Monotonicity in Production: $f(y,n) \geq f(\overline{y},n)$ for all $n \in Z_+^C$, and $y, \overline{y} \in \Re_+$ such that $y \geq \overline{y}$.

A *feasible state of the economy* is a list,

$$(X, Y, S) \equiv ((x_1, \ldots, x_I), (y^1, \ldots, y^P), (s^1, \ldots, s^P)),$$

where X is a list of private good for each agent, Y is a list of public good production plans for each jurisdiction, and S is a partition of the population, such that

$$\sum_{i=1}^{I} (\omega_{\tau(i)} - x_i) - \sum_{p=1}^{P} f(y^p, \mathcal{K}(s^p)) \geq 0.$$

The set containing all feasible states of the economy is denoted $\mathcal{F}$. Similarly, a pair (x, y) is a *feasible allocation for a jurisdiction* $s \in \mathcal{S}$ if,

$$\sum_{i \in s} (\omega_{\tau(i)} - x_i) - f(y^s, \mathcal{K}(s)) \geq 0.$$

A jurisdiction $\overline{s} \in \mathcal{S}$ producing a feasible allocation $(\overline{x}, \overline{y})$ *improves upon the state* $(X, Y, S) \in \mathcal{F}$ if, for all $i \in \overline{s}$ where $i \in s^p$ in the original feasible state,

$$u_{\tau(i)}(\overline{x}_i, \overline{y}, \mathcal{K}(\overline{s})) > u_{\tau(i)}(x_i, y^{s^p}, \mathcal{K}(s^p)).$$

A feasible state of the economy $(X, Y, S) \in \mathcal{F}$ is a *core state* if it cannot be improved upon by any jurisdiction.

Finally, the set of *optimal public good levels* for a jurisdiction $s \in \mathcal{S}$, denoted $Y(s)$, is

$$Y(s) \equiv \left\{ y \in \Re_+ \;\middle|\; y = \underset{y}{\operatorname{argmax}} \sum_{i \in s} \omega_{\mathcal{T}(i)} + \sum_{i \in s} h_{\mathcal{T}(i)}(y, \mathcal{K}(s)) - f(y, \mathcal{K}(s)) \right\}.$$

That is $y \in Y(s)$ maximizes total utility in jurisdiction s.

3. Cost Share Systems and Equilibria

A *cost share system* is a list, $\sigma = (\sigma_{11}, \ldots, \sigma_{ct}, \ldots, \sigma_{CT})$, stating for every $c \in \mathcal{C}$ and every $t \in \mathcal{T}$ a function

$$\sigma_{ct} : \Re_+ \times \mathcal{S}_{ct} \to \Re.$$

This cost share function for an agent of type (c, t) gives the total contribution necessary for this agent to be allowed into a jurisdiction s providing public goods level y. This is a very broad notion of cost-sharing analogous to definitions of cost sharing given by Mas-Colell (1980), Mas-Colell and Silvestre (1989), and Weber and Wiesmeth (1991). Note that at this point we have not required that the cost shares in a cost share system sum to total production costs. Although it is clear that in equilibrium jurisdictions must cover production costs, we might want different restrictions on cost shares defined over "potential" jurisdictions which do not appear in equilibrium partitions. For this reason, it is more straightforward to impose the feasibility requirement in the definition of a cost share equilibrium rather than in the definition of a cost share system.

Since tastes are not observable in this economy, finding cost shares which depend at most on an individual's crowding type and not on his or anyone else's taste type is of particular interest. Formally, a cost share system has *fully anonymous prices* (FAP) if it satisfies the following:

(FAP): for all $s, \hat{s} \in \mathcal{S}$, if $\mathcal{K}(s) = \mathcal{K}(\hat{s})$, then for all $y \in \Re_+$ and for all $t, \bar{t} \in \mathcal{T}$ it must be that $\sigma_{ct}(y, s) = \sigma_{c\bar{t}}(y, \hat{s})$.

This condition states that a fully anonymous cost share system is one for which any two individuals possessing the same crowding type and who reside in jurisdictions with identical crowding profiles must face the same cost shares. As is pointed out in Conley and Wooders (1997a), FAP is strong notion of price anonymity. A different interpretation of anonymity might allow two jurisdictions with identical crowding profiles to have different prices. In that case, anonymity would require only that the prices across jurisdiction be commonly available to agents with the same crowding characteristic but not necessarily the same tastes. Of course, using the stronger notion of price anonymity in this paper only stengthens the positive conclusion of Theorem 5.[6]

A *cost share equilibrium* consists of a feasible state of the economy $(X, Y, S) \in \mathcal{F}$ and a cost share system σ such that

1. for all $s^p \in S$, for all agents $i \in s^p$ where $\theta(i) = (c, t)$, all alternative jurisdictions $\bar{s} \in \mathcal{S}_c$, and all levels of public good $y \in \Re_+$,

$$\omega_t + h_t(y^p, \mathcal{K}(s^p)) - \sigma_{ct}(y^p, s^p) \geq \omega_t + h_t(y, \mathcal{K}(\bar{s})) - \sigma_{ct}(y, \bar{s}),$$

2. for all $s^p \in S$,

$$\sum_{i \in s^p} \sigma_{\kappa(i)\tau(i)}(y^p, s^p) - f(y^p, \mathcal{K}(s^p)) = 0,$$

3. for all alternative jurisdictions $\bar{s} \in \mathcal{S}$, and for all $y \in \Re_+$,

$$\sum_{i \in \bar{s}} \sigma_{\kappa(i)\tau(i)}(y, \bar{s}) - f(y, \mathcal{K}(\bar{s})) \leq 0.$$

Condition (1) requires that all agents maximize utility given the cost share system. Condition (2) requires that equilibrium jurisdictions exactly cover their production

[6] Also, the difference between the two interpretations of anonymity is not an issue in the two counterexamples that follow – both examples only consider prices *within* a single jurisdiction.

costs. Condition (3) requires that, given the cost share system, no firm can make positive profits by entering the market and offering to provide any sort of jurisdiction.[7]

If (X, Y, S) and σ constitute a cost share equilibrium, and σ is a fully anonymous cost share system, then we will say that (X, Y, S) and σ constitute an *anonymous cost share equilibrium.* We now consider how Tiebout admission price systems and Lindahl price systems fit within this broader notion of cost shares.

3.1 Tiebout Admission Prices

The Tiebout admission price system as defined in Conley and Wooders (1997a), for example, is a special class of the cost share systems. This class of cost share system states for each crowding type, for each jurisdiction, and for each public good level an "admission" price. That is, a *Tiebout admission price system* states for each $c \in \mathcal{C}$, a function

$$\sigma_c^A : \Re_+ \times \mathcal{S}_c \to \Re$$

which indicates the total contribution made by an agent of that type to enter a jurisdiction offering a fixed level of public goods and possessing a particular crowding profile. A *Tiebout equilibrium* is an anonymous cost share equilibrium in which the equilibrium cost shares are Tiebout admission prices. Conley and Wooders proved equivalence between the core and the set of Tiebout equilibrium states.[8] This result demonstrates that there is no need to require prices for public goods to depend on unobservable characteristics of agents and thus, is a type of Tiebout theorem. However, although the results applied to a broad class of economies (assumptions such as monotonicity, convexity, and continuity are not required), the Tiebout admission price system gen-

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[7] Sergui Hart pointed out in Conley and Wooders (1997a) that condition (2) is implied by condition (3) and feasibility. As before, condition (2) is maintained here to emphasize that jurisdiction formation is competitive in that equilibrium jurisdictions make zero profit.

[8] The core decentralization results is dependent upon the an assumption called *Small Group Effectiveness* which is a formalization of the fact that they consider a *local* public goods economy and not a pure public goods economy. This assumption is stated and discussed in a later section of this paper.

erally requires a continuum of prices — one price for every crowding type, for every possible jurisdiction, for every possible level of public good. For this reason, it seems unlikely that the Tiebout admission price system would ever be observed outside of a purely theoretical context.

3.2 Lindahl Prices

To find more widely applicable core decentralization results, the classic notion of a Lindahl equilibrium was modified for use in a crowding types economy. The Lindahl price system is a class of cost share system which specifies for each agent and for each jurisdiction a "participation" price and a "per-unit" price for consuming the local public good. That is, a *Lindahl price system* states for each $c \in \mathcal{C}$ and $t \in \mathcal{T}$ two functions,

$$a_{ct} : \mathcal{S}_{ct} \to \Re \quad \text{and} \quad \ell_{ct} : \mathcal{S}_{ct} \to \Re,$$

which combine to give a cost share function

$$\sigma^L_{ct}(s) \equiv a_{ct}(s) + \ell_{ct}(s)y.$$

Since the Lindahl prices do not depend on the level of public goods offered in a jurisdiction, Lindahl prices offer a finite-dimensional pricing space. Such a price system is anonymous or nonanonymous depending on whether it does or does not satisfy FAP, respectively. Conley and Wooders (1998) prove that for convex, monotonic and differentiable economies satisfying strict small group effectiveness, the set of nonanonymous Lindahl states is equivalent to the core and hence to the set of Tiebout admission price equilibrium states. They also show the set of anonymous Lindahl equilibria is generally smaller than the core – many core states cannot be decentralized with Lindahl prices satisfying FAP. In constrast to the larger class of economies which can be decentralized with anonymous Tiebout admission prices, only when there is exactly one crowding type and the technology exhibits constant returns to scale can core states generally be decentralized using anonymous Lindahl prices.

3.3 Finite Cost Shares

The main contribution of this paper is to explore another class of cost share systems we call finite cost shares. In this class of systems we hope to find decentralizing prices which are both finite dimensional and satisfy FAP. First we will provide a general definition of a finite cost share system, and then we will explore conditions under which they can decentralize the core.

A finite cost share system states for every type of agent a "participation price" and a "sharing ratio" of the cost of production. That is, a *finite cost share system* states for each $c \in \mathcal{C}$, for each $t \in \mathcal{T}$ two functions[9],

$$\alpha_{ct} : \Re_+ \times \mathcal{S}_{ct} \to \Re \quad \text{and} \quad \beta_{ct} : \Re_+ \times \mathcal{S}_{ct} \to \Re,$$

which combine to give a cost share function

$$\sigma^F_{ct}(y, s) \equiv \alpha_{ct}(y, s) + \beta_{ct}(y, s) f(y, \mathcal{K}(s)),$$

and where, for every $s \in \mathcal{S}$, the image sets over y of each $\alpha_{ct}(y, s)$ and $\beta_{ct}(y, s)$ are *finite sets.* Note that without this final restriction, we could clearly find fully anonymous cost shares that decentralize core states by setting each $\beta_{ct}(y, s) \equiv 0$ and then letting the $\alpha_{ct}(y, s)$ be the Tiebout admission prices. We say that the finite cost shares are *singled-valued* if, for each $s \in \mathcal{S}$, $\alpha_{ct}(y, s)$ and $\beta_{ct}(y, s)$ are both constant with respect to y.

4. Finite Cost Share Equilibrium and the Core

In this section, we explore the relationship between the core and the set of finite cost share states. Our first theorem states that anonymous and nonanonymous finite

[9] Both the participation price and the sharing ratio are contained in $\Re$, and no restriction is made on their signs.

cost share equilibria states are in the core. Combined with subsequent theorems showing conditions under which the core can be decentralized with various types of finite cost share prices, Theorem 1 implies a series of core equivalence results.[10]

Theorem 1. *If the state $(X, Y, S) \in \mathcal{F}$ and the finite cost share system σ^F constitute a finite cost share equilibrium, then (X, Y, S) is in the core.*

Proof/

See Appendix.

The next result is a First Welfare Theorem which is an immediate corollary of Theorem 1.

Theorem 2. *If the state $(X, Y, S) \in \mathcal{F}$ and σ^F constitute a finite cost share equilibrium then (X, Y, S) is Pareto optimal.*

Proof/

See Appendix.

We now restrict attention to economies in which all gains from coalition size are realized in small jurisdictions. An economy is said to satisfy *Strict Small Group Effectiveness* (SSGE) if there exists a positive integer B such that

1. for all core states (X, Y, S) and all $s^p \in S$, it holds that $|s^p| \leq B$;
2. for all $t \in \mathcal{T}$ and $c \in \mathcal{C}$, either $|\{i \in \mathcal{I} \mid \kappa(i) = c, \tau(i) = t\}| > B$ or $|\{i \in \mathcal{I} \mid \kappa(i) = c, \tau(i) = t\}| = 0$.

The first condition says that in all core states, agents are partitioned into jurisdictions bounded in size. The second condition is a thickness condition; either there are enough of a particular type of individual to fill the largest possible optimal jurisdiction, or no agent of that type appears in the economy at all.

The small group effectiveness assumption is one way to formalize the difference between a local public good economy and a pure public good economy. The primary

[10] For core decentralization results which do not restrict prices to be finite, refer to Conley and Wooders (1997a, 1998).

implication of SSGE is that the core jurisdictions will be small in comparison to the whole population. We now present two lemmas which are useful in proving the results that follow. The first lemma, a special case of Wooders (1983, Theorem 3) for games with strictly effective small groups, states that the core of an economy satisfying SSGE has the equal treatment property. The second lemma shows that the implicit contributions of all agents in a core jurisdiction are equal. These two lemmas are proved in Conley and Wooders (1997a).

Lemma 1. *Let (X, Y, S) be a core state of an economy satisfying SSGE. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\theta(i) = \theta(\hat{i}) = (c, t)$, if $i \in s^p$ and $\hat{i} \in s^{\hat{p}}$ then $u_t(x_i, y^p, \mathcal{K}(s^p)) = u_t(x_{\hat{i}}, y^{\hat{p}}, \mathcal{K}(s^{\hat{p}}))$.*

Lemma 2. *Let (X, Y, S) be a core state of an economy satisfying SSGE. If there is a core state jurisdiction $s^p \in \mathcal{S}_{ct} \cap \mathcal{S}_{c\hat{t}}$, then for any two individuals $i, \hat{i} \in s^p$ where $\theta(i) = (c, t)$ and $\theta(\hat{i}) = (c, \hat{t})$, it must be the case that $\omega_t - x_i = \omega_{\hat{t}} - x_{\hat{i}}$.*

In a pure public good framework, Weber and Wiesmeth (1991) show that there are upper and lower bounds on individuals' marginal rates of substitution such that only core collections satisfying those bounds can be decentralized by cost shares. In this local public good framework, we find similar conditions which must hold in order that a core allocation can be decentralized by finite cost shares. We first define an agent's arc-rate of substitution and explore its properties. We then state a bounding condition on the arc-rates of subtitution which allow core decentralization. Finally, we provide examples demonstrating why the bounding restriction is required.

Recall that $Y(s)$ defines the optimal levels of public goods for an arbitrary jurisdiction $s \in \mathcal{S}$. Let $y^s \in Y(s)$ and then for each $t \in \mathcal{T}$, define an individual's (normalized) *arc-rate of substitution* between y^s and any $y' \neq y^s$ as:

$$M_t(y', y^s, s) \equiv \frac{h_t(y^s, \mathcal{K}(s)) - h_t(y', \mathcal{K}(s))}{f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))}.$$

We first need to explore some properties of this function before stating a bounding condition needed to prove our primary results.

First, by monotonicity of $f(y,n)$ and each $h_t(y,n)$, it follows that each $M_t(y', y^s, s)$ is bounded below by 0 for $y' \neq y^s$. Second, by choice of y^s, we know that for all $y' \in \Re_+$,

$$\sum_{i\in s}\omega_{\tau(i)} + \sum_{i\in s} h_{\tau(i)}(y^s, \mathcal{K}(s)) - f(y^s, \mathcal{K}(s)) \geq$$

$$\sum_{i\in s}\omega_{\tau(i)} + \sum_{i\in s} h_{\tau(i)}(y', \mathcal{K}(s)) - f(y', \mathcal{K}(s)),$$

which immediately implies that for all $y' > y^s$,

$$\sum_{i\in s} M_{\tau(i)}(y', y^s, s) = \sum_{i\in s} \frac{h_{\tau(i)}(y^s, \mathcal{K}(s)) - h_{\tau(i)}(y', \mathcal{K}(s))}{f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))} \leq 1.$$

So since each $M_t(y', y^s, s) \geq 0$ and $\sum_{i\in s} M_{\tau(i)}(y', y^s, s) \leq 1$ for $y' > y^s$, it must be the case that each $M_t(y', y^s, s)$ is bounded above on $y' > y^s$. Therefore,

$$\sup_{y'>y^s} M_t(y', y^s, s) \quad \text{and} \quad \inf_{y'<y^s} M_t(y', y^s, s)$$

are well-defined functions for all $t \in \mathcal{T}$. We can now state a condition called *Bounded Aggregate Arc-Rate of Substitution* (BAARS) which we will need to decentralize core states with finite cost shares.

(BAARS): For all $s \in \mathcal{S}$,

$$\sum_{i\in s} \sup_{y'>y^s} M_{\tau(i)}(y', y^s, s) \ \leq 1 \leq \ \sum_{i\in s} \inf_{y'<y^s} M_{\tau(i)}(y', y^s, s).$$

Upon first glance, this condition may seem trivial. As the above development indicates, optimality of y^s requires that

$$\sum_{i\in s} M_{\tau(i)}(y', y^s, s) \ \leq 1$$

for any $y' > y^s$, and

$$\sum_{i\in s} M_{\tau(i)}(y', y^s, s) \geq 1$$

for any $y' < y^s$. That is, in the $y' > y^s$ case optimality requires that the sum of the agents' arc-rates of substitution for any y' is bounded above by 1, and in the $y' < y^s$

case optimality requires that the sume of the agents' arc-rates of substitution for any y' is bounded below by 1. But BAARS further requires that, in the first case, the sum across all agents of each agent's *largest* arc-rate of substitution is bounded above by 1.[11]

We now demonstrate that SSGE and BAARS are sufficient to allow the core to be decentralized with nonanonymous finite cost share prices. Combined with Theorem 1, this implies that the core and nonanonymous finite cost share equilibrium states are equivalent.

Theorem 3. *Let (X, Y, S) be a core state in an economy satisfying SSGE and BAARS. Then there exists a σ^F such that (X, Y, S) and σ^F constitute a nonanonymous finite cost share equilibrium.*

Proof/

See Appendix.

We now consider two examples that demonstrate the limits on the ability of finite cost shares to support core allocations. Example 1 shows that, in an economy which does not satisfy BAARS, core allocations may not be supportable by finite cost shares. The second example shows that, without further regularity conditions on cost and utility functions (recall that we have only assumed monotonicity), the finite cost shares that decentralize core allocations are not generally single-valued.

Example 1: Nonequivalence of the core and the finite cost share equilibria with differentiated crowding when the economy does not satisfy BAARS.

Consider a world with two crowding types and two taste types. Although generally this would mean that there are four types of people in the world, here we consider a differentiated crowding model in which the crowding types and the taste types are

[11] Note that this condition is apparently stronger than the bounded relative rates of substitution imposed on individuals preferences by Weber and Wiesmeth (1991). However, whereas Weber and Wiesmeth assume that preferences are convex, we make no assumptions about public good preferences other than monotonicity.

perfectly correlated. That is, there are only two types of people in the world, people with crowding type 1 and taste type 1, and people with crowding type 2 and taste type 2. This means that crowding is differentiated. Suppose that crowding occurs in consumption and that only two-person "mixed" jurisdictions get any utility from the public good. The utility functions of the two types are as follows:

$$u_1(x, y, n) = \begin{cases} x + y^2 & \text{for } y \leq 1 \text{ and if } n_{11} = n_{22} = 1, \\ x + 1 & \text{for } y > 1 \text{ and if } n_{11} = n_{22} = 1, \\ x & \text{otherwise} \end{cases}$$

$$u_2(x, y, n) = \begin{cases} x + y(2 - y) & \text{for } y \leq 1 \text{ and if } n_{11} = n_{22} = 1, \\ x + y & \text{for } y > 1 \text{ and if } n_{11} = n_{22} = 1, \\ x & \text{otherwise.} \end{cases}$$

Let the endowments of each type be zero: $\omega_1 = \omega_2 = 0$. The total cost of producing the public good is given by the function:

$$f(y, n) = 2y.$$

It is easily verified that $y = 1$ is the optimal public good level for any jurisdiction.

First we show that these utility functions do not satisfy BAARS. Note that for $\hat{s} = \{1, 2\}$

$$\inf_{y'<1} M_1(y', 1, \hat{s}) = \inf_{y'<1} \frac{1 + y'}{2} = \frac{1}{2}$$

and

$$\inf_{y'<1} M_2(y', 1, \hat{s}) = \inf_{y'<1} \frac{1 - y'}{2} = 0.$$

Therefore,

$$\sum_{i \in \hat{s}} \inf_{y'<1} M_{\tau(i)}(y', 1, \hat{s}) = \frac{1}{2},$$

which does not satisfy the second inequality in BAARS.

Now suppose there are 10 agents of each type in the economy. One of the core states will consists of ten two-person jurisdictions of the form $(x_i, x_j, y, s_1, s_2) = (-1, -1, 1, 1, 1)$. That is, each jurisdiction will contain one of each type of agent, and will produce one unit of public good. In this core state, the utilities of the two types of agents will be $U_1 = U_2 = 0$.

Notice that, since endowments are equal to zero, single-person jurisdictions get zero utility. Also, coalitions containing more than two people or any type of "non-mixed" coalition can get at most zero utility. Finally since the maximum total transferable utility available to each "mixed" jurisdiction is zero, such a coalition cannot improve upon itself by producing any other level of public good. Therefore, the state described above is a core state.

We now show that the above core state cannot be supported with finite cost shares. Begin by supposing that there are finite cost shares $(\alpha_{11}(y,s), \beta_{11}(y,s))$ and $(\alpha_{22}(y,s), \beta_{22}(y,s))$ which support this core state as a nonanonymous finite cost share equilibrium. Recall that here "finite" means that, for a given s, each $\alpha_{ct}(y,s)$ and $\beta_{ct}(y,s)$ take on finitely many values as y ranges over $\Re_+$. This means that, given the finite cost shares, we can partition $\Re_+$ into a finite number of subsets $\{Y^1, \dots, Y^q, \dots, Y^Q\}$ such that, for each $q = 1, \dots, Q$ and for any $y, \hat{y} \in Y^q$,

$$\alpha_{ct}(y,s) = \alpha_{ct}(\hat{y},s) \quad \text{and} \quad \beta_{ct}(y,s) = \beta_{ct}(\hat{y},s).$$

That is, we partition $\Re_+$ into finitely many subsets over which each $\alpha_{ct}(y,s)$ and each $\beta_{ct}(y,s)$ are constant with respect to y.

Now let Y^q be any set in the partition containing at least two elements.[12] For the rest of this example, we consider only the core state jurisdiction s and so we suppress this argument in the utility functions, cost functions, and cost share functions. Then by the assumption that $y = 1$ maximizes utility for each consumer given these finite cost shares, for all $y_1, y_2 \in Y^q$ and for any core state jurisdiction s,

$$h_1(1) - \alpha_{11}(1) - \beta_{11}(1)f(1) \geq h_1(y_1) - \alpha_{11}(y_1) - \beta_{11}(y_1)f(y_1)$$

$$h_1(1) - \alpha_{11}(1) - \beta_{11}(1)f(1) \geq h_1(y_2) - \alpha_{11}(y_1) - \beta_{11}(y_1)f(y_2).$$

Then rearranging these expressions we also have

$$\beta_{11}(y_1)f(y_1) \geq h_1(y_1) - h_1(1) + \alpha_{11}(1) - \alpha_{11}(y_1) + \beta_{11}(1)f(1)$$

[12] That such a set exists is guaranteed by the fact that we are finitely partitioning an infinite set. Therefore there must be an element of the partition containing an infinite number of elements.

and

$$\beta_{11}(y_1)f(y_2) \geq h_1(y_2) - h_1(1) + \alpha_{11}(1) - \alpha_{11}(y_1) + \beta_{11}(1)f(1).$$

It is easily verified that since $y_1 \neq y_2$, one of the above expressions must hold with *strict* inequality.[13] So without loss of generality, suppose that

$$\beta_{11}(y_1)f(y_1) > h_1(y_1) - h_1(1) + \alpha_{11}(1) - \alpha_{11}(y_1) + \beta_{11}(1)f(1).$$

Then by the assumption that $y = 1$ maximizes utility for the type 2 consumer given these cost shares,

$$\beta_{22}(y_1)f(y_1) \geq h_2(y_1) - h_1(1) + \alpha_{22}(1) - \alpha_{22}(y_1) + \beta_{22}(1)f(1).$$

Summing the above two inequalities we find

$$\begin{aligned}(\beta_{11}(y_1) + \beta_{22}(y_1))f(y_1) > h_1(y_1) - h_1(1) + h_2(y_1) - h_2(1) +\\ \alpha_{11}(1) + \alpha_{22}(1) - \alpha_{11}(y_1) - \alpha_{22}(y_2) + (\beta_{11}(1) + \beta_{22}(1))f(1) \qquad (*)\end{aligned}$$

Also by assumption, $y = 1$ maximizes profit for the core state jurisdiction s facing the above cost shares. That is,

$$\alpha_{11}(1) + \alpha_{22}(1) + (\beta_{11}(1) + \beta_{22}(1))f(1) - f(1) \geq$$

$$\alpha_{11}(y_1) + \alpha_{22}(y_1) + (\beta_{11}(y_1) + \beta_{22}(y_1))f(y_1) - f(y_1)$$

or, rearranging,

$$\begin{aligned}(\beta_{11}(y_1) + \beta_{22}(y_1))f(y_1) \leq & \alpha_{11}(1) + \alpha_{22}(1) - \alpha_{11}(y_1) - \alpha_{22}(y_1) +\\ & (\beta_{11}(1) + \beta_{22}(1))f(1) + f(y_1) - f(1) \qquad (**)\end{aligned}$$

We can then infer from equations $(*)$ and $(**)$ that

$$f(y_1) - f(1) > h_1(y_1) - h_1(1) + h_2(y_1) - h_2(1).$$

[13] This is not true in general. It holds only as a result of the utility function and cost function chosen in this example.

Substituting in the actual functions, this inequality reduces to $y_1 > y_1$, clearly a contradiction. Therefore, it must be the case that our chosen set Y^q does cannot contain two or more elements. However, since Y^q was chosen arbitrarily, this implies that no set in the partition contains more than one element. This is a contradiction to our supposition that we could partition $\Re_+$ into a finite number of sets by the cost shares $(\alpha_{11}(y,s), \beta_{11}(y,s))$ and $(\alpha_{22}(y,s), \beta_{22}(y,s))$. Therefore, these cost share functions cannot be finite. ∎

Example 2: Nonanonymous finite cost shares which decentralize core allocations in an economy satisfying SSGE and BAARS are not necessarily single-valued.

Consider a world with two crowding types and two taste types. Although generally this would mean that there are four types of people in the world, here we consider a differentiated crowding model in which the crowding types and the taste types are perfectly correlated. That is, there are only two types of people in the world, people with crowding type 1 and taste type 1, and people with crowding type 2 and taste type 2. This means that crowding is differentiated. Suppose that crowding occurs in consumption and the only two-person "mixed" jurisdictions get any utility from the public good. The utility functions of the two types are as follows:

$$u_1(x,y,n) = \begin{cases} x + y^2 + y & \text{for } y \le 1 \text{ and if } n_{11} = n_{22} = 1, \\ x + 2 & \text{for } y > 1 \text{ and if } n_{11} = n_{22} = 1, \\ x & \text{otherwise} \end{cases}$$

$$u_2(x,y,n) = \begin{cases} x + y(2-y) & \text{for } y \le 1 \text{ and if } n_{11} = n_{22} = 1, \\ x + y & \text{for } y > 1 \text{ and if } n_{11} = n_{22} = 1, \\ x & \text{otherwise.} \end{cases}$$

Let the endowments of each type be zero: $\omega_1 = \omega_2 = 0$. The total cost of producing the public good in jurisdiction s is given by the function:

$$f(y, \mathcal{K}(s)) = 2y.$$

Now suppose there are 10 agents of each type in the economy. One of the core states will consists of ten two-person jurisdictions of the form $(x_i, x_j, y, s^1, s^2) = (-1, -1, 1, 1, 1)$.

That is, each jurisdiction will contain one of each type of agent, and will produce one unit of public good. In this core state, the utilities of the two types of agents will be $U_1 = 1$ and $U_2 = 0$.

Notice that, since endowments are equal to zero, single-person jurisdictions get zero utility. Also, coalitions containing more than two people or any type of "non-mixed" coalition can get at most zero utility. Finally since the maximum total transferable utility available to each "mixed" jurisdictions is one, such coalitions cannot improve upon itself by producing any other level of public good. Therefore, the state described above is a core state.

It is easily verified that the finite cost shares defined in the proof of Theorem 3 decentralize this core state. Those cost shares are given by the following functions:

$$\alpha_{11}(y,s) = \begin{cases} -1 & \text{for } y \leq 1 \\ 1 & \text{for } y > 1 \end{cases} \quad \text{and} \quad \beta_{11}(y,s) = \begin{cases} 1 & \text{for } y \leq 1 \\ 0 & \text{for } y > 1 \end{cases}$$

$$\alpha_{22}(y,s) = \begin{cases} 1 & \text{for } y \leq 1 \\ 0 & \text{for } y > 1 \end{cases} \quad \text{and} \quad \beta_{22}(y,s) = \begin{cases} 0 & \text{for } y \leq 1 \\ 1/2 & \text{for } y > 1 \end{cases}$$

However, the purpose of this example is to demonstrate that it is impossible to decentralize this core state using single-valued cost shares. Let us suppose that, for some core state jurisdiction s, we can find decentralizing cost share functions $(\alpha_{11}(y,s), \beta_{11}(y,s))$ and $(\alpha_{22}(y,s), \beta_{22}(y,s))$ which do not vary with y.

Now consider the utility maximization problem for the type 2 agent in this jurisdiction. Utility maximization for the type 2 agent at $y = 1$ in s given α_{11} and β_{11} requires that[14]

$$h_2(1) - \alpha_{22} - \beta_{22} f(1) \geq h_2(y) - \alpha_{22} - \beta_{22} f(y),$$

for all y. However, substituting in the utility and cost functions from above, this expression can only be satisfied for all $y < 1$ if $\beta_{22} \leq 0$ and can only be satisfied for all $y > 1$ if $\beta_{22} \geq 1/2$. Clearly, this means that any decentralizing cost shares cannot be constant over all y. ■

[14] Note here that we are suppressing the arguments of the cost-share functions.

Our next result says that if utility and production functions are differentiable and satisfy convexity, and the economy satisfies SSGE, then the core can be decentralized by *single-valued* nonanonymous finite cost share prices.

Theorem 4. *Let $h_t(y, n)$ be differentiable and concave in y for all $t \in \mathcal{T}$ and let $f(y, n)$ be differentiable and convex in y. If (X, Y, S) is a core state in an economy satisfying SSGE, then there exists a finite cost share system σ^F such that (X, Y, S) and σ^F constitute a finite cost share equilibrium, and each $\sigma^F_{ct}(y, s)$ is independent of y.*

Proof/

See Appendix.

We are now prepared to state our main result. Wooders (1978) proved that in an economy with strictly small effective groups and only one crowding type, if technology exhibits constant returns to scale, then core allocations can be anonymously decentralized with Lindahl prices. The following theorem shows that it is possible to decentralize core allocations with anonymous and finite prices *without* the restrictive assumption of constant returns to scale. However, anonymous decentralization requires a strengthening of our BAARS hypothesis.

The economy is said to have a *Strongly Bounded Aggregate Arc-Rate of Substitution* (SBAARS) if for all $s \in \mathcal{S}$,

$$|s| \max_{i \in s} \sup_{y' > y^s} M_{\tau(i)}(y', y^s, s) \ \leq 1 \leq \ |s| \min_{i \in s} \inf_{y' < y^s} M_{\tau(i)}(y', y^s, s).$$

Theorem 5. *Let (X, Y, S) be a core state in an economy satisfying SSGE and SBAARS and with only one crowding type. Then there exists a σ^F such that (X, Y, S) and σ^F constitute an anonymous finite cost share equilibrium.*

Proof/

See Appendix.

To see that Theorem 5 is a more general result than the existing anonymous decentralization results, we now present two addition examples of economies (one with differentiated crowding and one with anonymous crowding) which satisfy SBAARS but which do not have linear technologies.

Note that the economy presented in Example 2 satisfies SBAARS and agents have convex preferences. But perhaps that example is less interesting with respect to how the SBAARS condition relates to the extant literature because that economy also has linear technology – the value of the SBAARS restriction is that it allows for anonymous core decentralization without restriction to linear technologies. Therefore, we present the following example of an economy satisfying SBAARS but without linear technology.

Example 3: An economy satisfying SSGE and SBAARS without linear technology.

Consider an economy consisting of exactly the same agents, population, and endowments as in Example 2, but with technology as follows:

$$f(y, \mathcal{K}(s)) = \begin{cases} 2y & \text{for } y \leq 1, \\ 1 + y^2 & \text{for } y > 1 \end{cases}.$$

Just as in Example 2, one of the core states will consists of ten two-person jurisdictions of the form $(x_i, x_j, y, s^1, s^2) = (-1, -1, 1, 1, 1)$. That is, each jurisdiction will contain one of each type of agent, and will produce one unit of public good. In this core state, the utilities of the two types of agents will be zero. Showing this economy satisfies SBAARS is a straightforward calculation following the procedure in Example 2.

Perhaps a more interesting example of an economy satisfying SBAARS without linear technology is one which bears directly on Theorem 5, an economy with only one crowding type in which core states can be decentralized with anonymous finite cost shares but which cannot be decentralized with anonymous Lindahl prices. Example 4 presents such an economy.

Example 4: An economy satisfying SSGE and SBAARS with one crowding type and without linear technology.

Consider a world consisting of an even number of identical individuals with the following utility function:

$$u(x,y,n)=\begin{cases} x+y^2+y & \text{for } y\le 1 \text{ and if } |n|=2, \\ x+2 & \text{for } y>1 \text{ and if } n_{11}=n_{22}=1, \\ x & \text{otherwise} \end{cases} .$$

Let the endowments of each person be zero and he total cost of producing the public good in jurisdiction s is given by the function:

$$f(y,\mathcal{K}(s))=\begin{cases} 2y & \text{for } y\le 1, \\ 1+y^2 & \text{for } y>1 \end{cases} .$$

Core states will consists of two-person jurisdictions of the form $(x_1,x_2,y,s^1,s^2)=(-1,-1,1,1,1)$. That is, each jurisdiction will contain two agents receiving -1 units of private good, and will produce one unit of public good. In this core state, the utility of each agent will be zero. Again, showing this economy satisfies SBAARS is a straightforward calculation following the procedure in Example 2.

5. Conclusions

The point of this paper was to extend the domain of local public goods economies for which it is possible to decentralize the core with a finite and anonymous price system.[15] Previously, it was only known that such decentralization was possible with Lindahl prices when there was one crowding type and technology was linear. We define a type of price system called finite cost shares. This includes Lindahl prices as a special case but is a strict subset of all possible cost share price systems. We show that if there is only one crowding type, preferences and production functions are monotonic, and a bounding condition on normalized arc-rates of substitution called SBAARS is satisfied, then *anonymous* finite cost shares decentralize the core. Thus, no convexity

[15] We maintain the assumption of strict small group effectiveness for all of our discussion of local public goods economies. This is because SSGE is precisely what implies that efficiency can only be achieved in the economy when agents break up in a system of jurisdictions that are small compared the population. Thus, SSGE is one way of defining exactly what a local public goods economy is.

or differentiability of preferences or production functions is required. We also show that with more than one crowding type, *nonanonymous* decentralization of the core is possible and that SBAARS can be replaced by a weaker condition called BAARS.

The above equivalence results implies that the questions of existence of a finite cost-share equilibrium and core existence are the same question. While it has long been understood that the core is often empty in these local public goods economies – see Pauly (1970) and Wooders (1978) for early discussions – the ϵ-core can be shown to exist (see Wooders (1980) and the exact core exists in continuum versions (see Cole and Prescott (1997), Conley and Wooders (1997b), and Ellickson, *et al.* (1999)).

Finally, the core is a cooperative notion and, as is the tradition in this literature, our decentralization results show that this stable allocation can be supported by fulling decentralizing prices. Although implementation is an interesting question, it is clearly beyond the scope of this work.

Appendix

Theorem 1. *If the state $(X, Y, S) \in \mathcal{F}$ and the finite cost share system σ^F constitute a finite cost share equilibrium, then (X, Y, S) is in the core.*

Proof/

Suppose that (X, Y, S) and σ^F constitute a finite cost share equilibrium but (X, Y, S) is not in the core. Then there exists a jurisdiction, $\hat{s} \in \mathcal{S}$, and an allocation, $(\hat{x}, \hat{y})$, such that

$$\sum_{i \in \hat{s}} \omega_{\tau(i)} - \sum_{i \in \hat{s}} \hat{x}_i - f(\hat{y}, \mathcal{K}(\hat{s})) \geq 0 \tag{1.1}$$

and for all $i \in \hat{s}$ where (X, Y, S) assigns $i \in s^{p_i}$ and allocation (x_i, y^{p_i}),

$$u_{\tau(i)}(\hat{x}_i, \hat{y}, \mathcal{K}(\hat{s})) > u_{\tau(i)}(x_i, y^{p_i}, \mathcal{K}(s^{p_i}).$$

Expanding this and summing over all $i \in \hat{s}$ gives

$$\sum_{i \in \hat{s}} \hat{x}_i + \sum_{i \in \hat{s}} h_{\tau(i)}(\hat{y}, \mathcal{K}(\hat{s})) > \sum_{i \in \hat{s}} x_i + \sum_{i \in \hat{s}} h_{\tau(i)}(y^{p_i}, \mathcal{K}(s^{p_i})). \tag{1.2}$$

Now if agent $i \in \hat{s}$ were to pay the finite cost shares assigned by σ^F for jurisdictions s^{p_i} and $\hat{s}$, by definition the private goods consumption of agent i in these two jurisdictions would be

$$x_i \equiv \omega_{\tau(i)} - \alpha_{\kappa(i)\tau(i)}(y^{p_i}, s^{p_i}) - \beta_{\kappa(i)\tau(i)}(y^{p_i}, s^{p_i}) f(y^{p_i}, \mathcal{K}(s^{p_i}))$$

$$\text{and} \tag{1.3}$$

$$\tilde{x}_i \equiv \omega_{\tau(i)} - \alpha_{\kappa(i)\tau(i)}(\hat{y}, \hat{s}) - \beta_{\kappa(i)\tau(i)}(\hat{y}, \hat{s}) f(\hat{y}, \kappa(\hat{s})).$$

Note that $\tilde{x}_i$ is not necessarily the allocation assigned by the improving allocation $\hat{x}$. By utility maximization under σ^F we know that

$$\omega_{\tau(i)} - \alpha_{\kappa(i)\tau(i)}(y^{p_i}, s^{p_i}) - \beta_{\kappa(i)\tau(i)}(y^{p_i}, s^{p_i}) f(y^{p_i}, \mathcal{K}(s^{p_i})) + h_{\tau(i)}(y^{p_i}, \mathcal{K}(s^{p_i})) \geq$$

$$\omega_{\tau(i)} - \alpha_{\kappa(i)\tau(i)}(\hat{y}, \hat{s}) - \beta_{\kappa(i)\tau(i)}(\hat{y}, \hat{s}) f(\hat{y}, \mathcal{K}(\hat{s})) + h_{\tau(i)}(\hat{y}, \mathcal{K}(\hat{s})). \tag{1.4}$$

Substituting x_i and $\tilde{x}_i$ from (1.3) into (1.4) and summing over all $i \in \hat{s}$ gives

$$\sum_{i \in \hat{s}} x_i + \sum_{i \in \hat{s}} h_{\tau(i)}(y^{p_i}, \mathcal{K}(s^{p_i})) \geq \sum_{i \in \hat{s}} \tilde{x}_i + \sum_{i \in \hat{s}} h_{\tau(i)}(\hat{y}, \mathcal{K}(\hat{s})).$$

This implies, directly from (1.2), that

$$\sum_{i \in \hat{s}} \hat{x}_i + \sum_{i \in \hat{s}} h_{\tau(i)}(\hat{y}, \mathcal{K}(\hat{s})) > \sum_{i \in \hat{s}} \tilde{x}_i + \sum_{i \in \hat{s}} h_{\tau(i)}(\hat{y}, \mathcal{K}(\hat{s}))$$

or

$$\sum_{i \in \hat{s}} \hat{x}_i > \sum_{i \in \hat{s}} \tilde{x}_i. \tag{1.5}$$

However, by profit maximization under σ^F we know that

$$\sum_{i \in \hat{s}} \omega_{\tau(i)} - \sum_{i \in \hat{s}} \tilde{x}_i - f(\hat{y}, \mathcal{K}(\hat{s})) \leq 0$$

or

$$\sum_{i \in \hat{s}} \omega_{\tau(i)} - f(\hat{y}, \mathcal{K}(\hat{s})) \leq \sum_{i \in \hat{s}} \tilde{x}_i$$

which from (1.1) reduces to

$$\sum_{i \in \hat{s}} \hat{x}_i \leq \sum_{i \in \hat{s}} \tilde{x}_i,$$

a contradiction to (1.5). ∎

Theorem 2. *If the state $(X, Y, S) \in \mathcal{F}$ and σ^F constitute a finite cost share equilibrium then (X, Y, S) is Pareto optimal.*

Proof/

By Theorem (1), the set of finite cost share states are contained in the set of core states. Since all core states are Pareto optimal, so are all finite cost share states. ∎

Theorem 3. *Let (X, Y, S) be a core state in an economy satisfying SSGE and BAARS. Then there exists a σ^F such that (X, Y, S) and σ^F constitute a nonanonymous finite cost share equilibrium.*

Proof/

Since the economy satisfies SSGE, by Lemma 1 all agents of the same taste and crowding type are treated equally in any core state, irrespective of their assigned jurisdiction. Let U_{ct} denote the utility level received in the core state by agents with type (c, t). Then for all $c \in \mathcal{C}$ and all $t \in \mathcal{T}$, denote the total willingness of an agent of type (c, t) to pay to join a jurisdiction $s \in \mathcal{S}_{ct}$ offering $y \in \Re_+$ public goods as:

$$twp_{ct}(y, s) \equiv \omega_t + h_t(y, \mathcal{K}(s)) - U_{ct}, \tag{3.1}$$

i.e., the total willingness to pay is the surplus utility received in s over the core state utility.

Since (X, Y, S) is a core state, in each core state jurisdiction, $s^1, \ldots, s^P$, the corresponding levels of public goods, $y^1, \ldots, y^P$, must maximize total utility. That is, $y^p \in Y(s^p)$ for $p = 1, \ldots, P$. In each alternative jurisdiction $s \in \mathcal{S} - S$, let y^s be a fixed but arbitrary element of $Y(s)$.

For each $t \in \mathcal{T}$ and for each $s \in \mathcal{S}$, define the finite cost share ratios as follows:

$$\beta_t(y, s) \equiv \begin{cases} \inf\limits_{y' < y^s} M_t(y', y^s, s), & \text{if } y < y^s; \\ \sup\limits_{y' > y^s} M_t(y', y^s, s), & \text{if } y \geq y^s. \end{cases} \tag{3.2}$$

As state above, since each $M_t(y', y^s, s)$ is bounded below and is bounded above on $y' > y^s$, the function $\beta_t(y, s)$ is well-defined. However, generally $M_t(y', y^s, s)$ is neither continuous nor monotonically decreasing and so, for a given s, the image $\beta_t(y, s)$ generally contains two elements. Now for each $c \in \mathcal{C}$, $t \in \mathcal{T}$, and each $s \in \mathcal{S}_{ct}$, define the finite cost share participation component as follows:

$$\alpha_{ct}(y, s) \equiv twp_{ct}(y^s, s) - \beta_t(y, s) f(y^s, \mathcal{K}(s)). \tag{3.3}$$

Note that for each s the image set of each $\alpha_{ct}(y, s)$ generally contains two elements because each $\beta_t(y, s)$ does.

1. We begin by showing that when agents maximize utility given these finite cost shares they can do no better than the jurisdictions to which they are assigned by the core state partition. Given these cost shares, an agent i of type (c, t) consuming $y \in \Re_+$ public goods in any jurisdiction $s \in \mathcal{S}_{ct}$ gets utility

$$\omega_t + h_t(y, \mathcal{K}(s)) - \alpha_{ct}(y, s) - \beta_t(y, s) f(y, \mathcal{K}(s)).$$

Substituting in $\alpha_{ct}(y, s)$ from (3-3-) and then $twp_{ct}(y, s)$ from (3.1), the agent's utility is

$$\omega_t + h_t(y, \mathcal{K}(s)) - [twp_{ct}(y^s, s) - \beta_t(y, s) f(y^s, \mathcal{K}(s))] - \beta_t(y, s) f(y, \mathcal{K}(s))$$

$$= U_{ct} + h_t(y, \mathcal{K}(s)) - h_t(y^s, \mathcal{K}(s)) + \beta_t(y, s)[f(y^s, \mathcal{K}(s)) - f(y, \mathcal{K}(s))].$$

Clearly, given the defined cost shares, agent i gets U_{ct} by choosing y^s public goods in jurisdiction s. So it remains to show for all $y \in \Re_+$ that

$$h_t(y, \mathcal{K}(s)) - h_t(y^s, \mathcal{K}(s)) + \beta_t(y, s)[f(y^s, \mathcal{K}(s)) - f(y, \mathcal{K}(s))] \leq 0. \tag{3.4}$$

But by definition of $\beta_t(y, s)$, for all $y < y^s$

$$\beta_t(y, s) \leq \frac{h_t(y^s, \mathcal{K}(s)) - h_t(y, \mathcal{K}(s))}{f(y^s, \mathcal{K}(s)) - f(y, \mathcal{K}(s))} \tag{3.5}$$

and for all $y > y^s$

$$\beta_t(y, s) \geq \frac{h_t(y^s, \mathcal{K}(s)) - h_t(y, \mathcal{K}(s))}{f(y^s, \mathcal{K}(s)) - f(y, \mathcal{K}(s))}. \tag{3.6}$$

Therefore, for all $y \in \Re_+$

$$\beta_t(y, s)[f(y^s, \mathcal{K}(s)) - f(y, \mathcal{K}(s))] \leq h_t(y^s, \mathcal{K}(s)) - h_t(y, \mathcal{K}(s)),$$

which is precisely equation (3.4). This shows that in all possible alternative jurisdictions and all possible levels of public good, given this cost share system, agents can do no better that their assigned core state jurisdictions and the optimal level of public good for that jurisdiction.

2. Next we show that the jurisdictions in the core partition generate enough revenue at the constructed cost shares to pay for the public good level they provide. Given the constructed cost shares, total revenues in core state jurisdiction s^p are

$$\sum_{i \in s^p} \alpha_{\kappa(i)\tau(i)}(y^p, s^p) + \sum_{i \in s^p} \beta_{\tau(i)}(y^p, s^p) f(y^p, \mathcal{K}(s^p)) \tag{3.7}$$

Now substituting in each $twp_{ct}(y^p, s^p)$ from (3.1) into (3.7), revenues in s^p reduce to

$$\sum_{i \in s^p} \omega_{\tau(i)} + \sum_{i \in s^p} h_{\tau(i)}(y^p, \mathcal{K}(s^p)) - \sum_{i \in s^p} U_{\kappa(i)\tau(i)}.$$

But since core state jurisdictions are feasible, we know that

$$\sum_{i \in s^p} \omega_{\tau(i)} + \sum_{i \in s^p} h_{\tau(i)}(y^p, \mathcal{K}(s^p)) - \sum_{i \in s^p} U_{\kappa(i)\tau(i)} = f(y^p, \mathcal{K}(s^p)).$$

Therefore, revenues exactly cover costs in core state jurisdictions under σ^F.

3. Now we show that no jurisdiction s can do any better than providing y^s public goods given the finite cost share system. Jurisdiction s maximizes profit producing y^s given σ^F if and only if for all $y \in \Re_+$

$$\sum_{i \in s} \alpha_{\kappa(i)\tau(i)}(y^s, s) + \sum_{i \in s} \beta_{\tau(i)}(y^s, s) f(y^s, \mathcal{K}(s)) - f(y^s, \mathcal{K}(s)) \geq$$

$$\sum_{i\in s}\alpha_{\kappa(i)\tau(i)}(y,s)+\sum_{i\in s}\beta_{\tau(i)}(y,s)f(y,\mathcal{K}(s))-f(y,\mathcal{K}(s)). \quad (3.8)$$

By construction,

$$\alpha_{ct}(y^s,s)\equiv twp_{ct}(y^s,s)-\beta_t(y^s,s)f(y^s,\mathcal{K}(s)),$$

and

$$\alpha_{ct}(y,s)\equiv twp_{ct}(y^s,s)-\beta_t(y,s)f(y^s,\mathcal{K}(s)),$$

Thus substituting in $\alpha_{ct}(y^s,s)$ and $\alpha_{ct}(y,s)$ into (3.8), and collecting like terms, (3.8) becomes

$$[1-\sum_{i\in s}\beta_{\tau(i)}(y,s)][f(y^s,\mathcal{K}(s))-f(y,\mathcal{K}(s))]\leq 0.$$

This implies that profits are maximized at y^s if and only if

$$\sum_{i\in s}\beta_{\tau(i)}(y,s)\geq 1 \text{ for all } y<y^s \quad (3.9)$$

and

$$\sum_{i\in s}\beta_{\tau(i)}(y,s)\leq 1 \text{ for all } y>y^s. \quad (3.10)$$

But by the construction of $\beta_t(y,s)$, and by BAARS, this condition is satisfied. Therefore jurisdiction s maximizes profits at y^s.

4. Finally, we show that no alternative jurisdiction can do better than core state jurisdictions by showing that they cannot have positive profits. Recall that core state jurisdictions exactly cover costs under these cost shares, and the above argument shows that all jurisdictions maximize profit at y^s. The total profit in jurisdiction any s providing y^s public goods is

$$\sum_{i\in s}\alpha_{\kappa(i)\tau(i)}(y^s,s)+\sum_{i\in s}\beta_{\kappa(i)\tau(i)}(y^s,s)f(y^s,\mathcal{K}(s))-f(y^s,\mathcal{K}(s)). \quad (3.11)$$

By construction (see equations (3.3) and (3.1)),

$$\alpha_{\kappa(i)\tau(i)}(y^s,s)=\omega_{\tau(i)}+h_{\tau(i)}(y^s,\mathcal{K}(s))-U_{\kappa(i)\tau(i)}-\beta_{\tau(i)}(y^s,s)f(y^s,\mathcal{K}(s)). \quad (3.12)$$

Summing (3.12) over all agents gives

$$\sum_{i\in s}\alpha_{\kappa(i)\tau(i)}(y^s,s)=$$

$$\sum_{i\in s}\omega_{\tau(i)}+\sum_{i\in s}h_{\tau(i)}(y^s,\mathcal{K}(s))-\sum_{i\in s}U_{\kappa(i)\tau(i)}-\sum_{i\in s}\beta_{\tau(i)}(y^s,s)f(y^s,\mathcal{K}(s)).$$

Thus profit in jurisdiction s, (3.11), reduces to

$$\sum_{i\in s}\omega_{\tau(i)}+\sum_{i\in s}h_{\tau(i)}(y^s,\mathcal{K}(s))-f(y^s,\mathcal{K}(s))-\sum_{i\in s}U_{\kappa(i)\tau(i)}.$$

Suppose for some alternative jurisdiction $\bar{s}\in\mathcal{S}$ that profits were strictly positive. But then

$$\sum_{i\in\bar{s}}U_{\kappa(i)\tau(i)}<\sum_{i\in\bar{s}}\omega_{\tau(i)}+\sum_{i\in\bar{s}}h_{\tau(i)}(y^{\bar{s}},\mathcal{K}(\bar{s}))-f(y^{\bar{s}},\mathcal{K}(\bar{s})),$$

and the agents in $\bar{s}$ could do better than they do in their assigned core state jurisdiction by producing $y^{\bar{s}}$ and distributing the surplus private good. This contradicts the hypothesis that (X,Y,S) is a core state. Thus no jurisdiction can make positive profit under these prices. ■

Theorem 4. *Let $h_t(y,n)$ be differentiable and concave in y for all $t\in\mathcal{T}$ and let $f(y,n)$ be differentiable and convex in y. If (X,Y,S) is a core state in an economy satisfying SSGE, then there exists a finite cost share system σ^F such that (X,Y,S) and σ^F constitute a finite cost share equilibrium, and each $\sigma^F_{ct}(y,s)$ is independent of y.*

Proof/

We prove this result by showing that, under these hypotheses, BAARS is satisfied and the finite cost shares constructed in Theorem (3) are independent of y.

To show that each $\beta_t(y,s)$ is single-valued for a given s, consider again the function

$$M_t(y',y^s,s)\equiv\frac{h_t(y^s,\mathcal{K}(s))-h_t(y',\mathcal{K}(s))}{f(y^s,\mathcal{K}(s))-f(y',\mathcal{K}(s))},$$

defined everywhere on $\Re_+$ except y^s. By l'Hôpital's rule,

$$\lim_{y'\to y^s}M_t(y',y^s,s)=\lim_{y'\to y^s}\frac{h_t'(y',\mathcal{K}(s))}{f'(y',\mathcal{K}(s))}=\frac{h_t'(y^s,\mathcal{K}(s))}{f'(y^s,\mathcal{K}(s))}.$$

This means that the limits from the left of y^s and from the right of y^s are the same. Thus to show that $\beta_t(y,y^s,s)$ is single-valued, it suffices to show that $M_t(y',y^s,s)$ is decreasing on $\Re_+$ (except y^s) because that would imply that

$$\inf_{y'<y^s}M_t(y',y^s,s)=\sup_{y'>y^s}M_t(y',y^s,s). \tag{4.1}$$

But $M_t(y',y^s,s)$ is differentiable everywhere on $\Re_+$ except y^s and it is easily verified that $M_t'(y',y^s,s)\leq 0$ at y' if and only if

$$f'(y',\mathcal{K}(s))[h_t(y^s,\mathcal{K}(s))-h_t(y',\mathcal{K}(s))]\leq h_t'(y',\mathcal{K}(s))[f(y^s,\mathcal{K}(s))-f(y',\mathcal{K}(s))]. \tag{4.2}$$

However, convexity of $f(y, \mathcal{K}(s))$ requires that

$$f'(y', \mathcal{K}(s)) \le \frac{f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))}{y^s - y'} \quad \text{for all } y' < y^s, \tag{4.3}$$

and concavity of $h_t(y, \mathcal{K}(s))$ requires that

$$\frac{h_t(y^s, \mathcal{K}(s)) - h_t(y', \mathcal{K}(s))}{y^s - y'} \le h'_t(y', \mathcal{K}(s)) \quad \text{for all } y' < y^s. \tag{4.4}$$

Multiplying equations (4.3) and (4.4) gives, for all $y' < y^s$,

$$f'(y', \mathcal{K}(s)) \frac{[h_t(y^s, \mathcal{K}(s)) - h_t(y', \mathcal{K}(s))]}{y^s - y'} \le \frac{h'_t(y', \mathcal{K}(s))[f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))]}{y^s - y'}. \tag{4.5}$$

Then multiplying (4.5) by $y^s - y'$ (which is positive, in this case) gives (4.2). Similarly, convexity of $f(y, \mathcal{K}(s))$ requires that

$$f'(y', \mathcal{K}(s)) \ge \frac{f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))}{y^s - y'} \quad \text{for all } y' > y^s \tag{4.6}$$

and concavity of $h_t(y, \mathcal{K}(s))$ implies that

$$\frac{h_t(y^s, \mathcal{K}(s)) - h_t(y', \mathcal{K}(s))}{y^s - y'} \ge h'_t(y', \mathcal{K}(s)) \quad \text{for all } y' > y^s. \tag{4.7}$$

Multiplying (4.6) and (4.7) gives, for all $y' > y^s$,

$$f'(y', \mathcal{K}(s)) \frac{[h_t(y^s, \mathcal{K}(s)) - h_t(y', \mathcal{K}(s))]}{y^s - y'} \ge \frac{h'_t(y', \mathcal{K}(s))[f(y^s, \mathcal{K}(s)) - f(y', \mathcal{K}(s))]}{y^s - y'}. \tag{4.8}$$

And then multiplying (4.8) by $y^s - y'$ (which is negative, in this case) gives (4.2). Therefore, (4.2) is satisfied for all $y' \neq y^s$ and hence $M'_t(y', y^s, s) \le 0$ for all $y' \neq y^s$. As indicated above, this proves that $\beta_t(y, s)$ is independent of y for all $t \in \mathcal{T}$. It follows immediately that each $\alpha_{ct}(y, s)$ is also independent of y. ■

Theorem 5. *Let (X, Y, S) be a core state in an economy satisfying SSGE and SBAARS and with only one crowding type. Then there exists a σ^F such that (X, Y, S) and σ^F constitute an anonymous finite cost share equilibrium.*

Proof/

Since the economy has only one crowding type, we will drop all indexes referring to an agent's crowding characteristic. Also note that with only one crowding type, the crowding profile of a jurisdiction depends only on the total number of agents in the jurisdiction.

By Lemma 1, identical agents are treated equally in the core. Let U_t denote the core state utility of a type $t \in \mathcal{T}$ agent. Just as in the proof for Theorem 3, let y^s denote the core state public good levels for all $s \in S$ and an arbitrary element of $Y(s)$ for all other jurisdictions. First, define finite cost share ratios as in Theorem 3:

$$\beta_t(y,s) \equiv \begin{cases} \inf_{y'<y^s} M_{\tau(i)}(y',y^s,s), & \text{if } y < y^s; \\ \sup_{y'>y^s} M_{\tau(i)}(y',y^s,s), & \text{if } y \geq y^s. \end{cases}$$

As in Theorem (3), each $M_t(y, y^s, s)$ is bounded in the appropriate ways to make $\beta_t(y, s)$ a well-defined function. Next, define new a new finite cost share ratio which doesn't depend on type as follows:

$$\beta(y,s) \equiv \begin{cases} \min_{i \in s} \beta_t(y,s), & \text{if } y < y^s; \\ \max_{i \in s} \beta_t(y,s), & \text{if } y \geq y^s. \end{cases}$$

Finally, define each finite cost share participation component as follows:

$$\alpha_t(y,s) \equiv twp_t(y^s,s) - \beta(y,s) f(y^s, \mathcal{K}(s)).$$

Note that for all $y < y^s$ and for all $t \in \mathcal{T}$,

$$\beta(y,s) \leq \beta_t(y,s);$$

and for all $y \geq y^s$ and for all $t \in \mathcal{T}$,

$$\beta(y,s) \geq \beta_t(y,s).$$

From here the arguments which show that (1) all agents maximize utility at y^s under these cost shares and can do no better than their assigned jurisdictions; (2) production costs are exactly covered in core state jurisdictions; and (3) jurisdictions maximize profit in any jurisdiction at y^s and no alternative jurisdiction can do better than the core state jurisdictions are formally identical to the same arguments in the prove of Theorem 3. To see why the stronger restriction of SBAARS is needed, consider that anonymous cost shares requires that each the $\beta(y, s)$ need to be identical and yet still satisfy equations (3.5) and (3.6). Hence $\beta(y, s)$, the common and identical cost sharing ratio, must be as small as the smallest $\beta_t(y, s)$ for all $y < y^s$ and as large as the largest $\beta_t(y, s)$ for all $y \geq y^s$ in order to satisfy utility-maximization for all consumers. But then profit maximization given by equations (3.9) and (3.10) becomes

$$|s| \min_{i \in s} \beta_{\tau(i)}(y,s) \geq 1 \;\; \text{for all} y < y^s$$

and

$$|s| \max_{i \in s} \beta_{\tau(i)}(y,s) \leq 1 \;\; \text{for all} y > y^s.$$

However, this is satisfied exactly by our stronger assumption, SBAARS.

What remains to be shown is that the cost share functions defined here satisfy FAP. Clearly, $\beta(y, s)$ is the same across all types *within* a jurisdiction. And by Lemma (2), all implicit contributions to public goods are anonymous. Thus, in any core state jurisdiction s^p, for all $i, \hat{i} \in s^p$ where $\tau(i) = t$ and $\tau(\hat{i}) = \hat{t}$,

$$\begin{aligned}\omega_t - x_i =& \alpha_t(y^p, s^p) + \beta(y^p, s^p) f(y^p, \mathcal{K}(s^p)) = \\ & \alpha_{\hat{t}}(y^p, s^p) + \beta(y^p, s^p) f(y^p, \mathcal{K}(s^p)) = \omega_{\hat{t}} - x_{\hat{i}}.\end{aligned}$$

We can therefore conclude that all agents in a core state jurisdiction pay identical participation prices and cost share ratios in their assigned jurisdictions. We now show that the cost share system satisfies FAP. That is, we show $\sigma_t^F(y^s, s) = \sigma_t^F(y^{\hat{s}}, \hat{s})$ whenever $y^s = y^{\hat{s}}$ and $\mathcal{K}(s) = \mathcal{K}(\hat{s})$.

Consider any two jurisdictions $s, \hat{s} \in \mathcal{S}$ satisfying $\mathcal{K}(s) = \mathcal{K}(\hat{s})$. Note that since there is only one crowding type, this means we consider any two jurisdictions $s, \hat{s} \in \mathcal{S}$ satisfying $|s| = |\hat{s}|$. Then, by construction, for any $i \in \mathcal{I}$ where $\tau(i) = t$,

$$\begin{aligned}\sigma_t^F(y^s, s) &= \alpha_t(y^s, s) + \beta(y^s, s) f(y^s, \mathcal{K}(s)) \\ &= \omega_t + h_t(y^s, \mathcal{K}(s)) - U_t\end{aligned}$$

and

$$\begin{aligned}\sigma_t^F(y^{\hat{s}}, \hat{s}) &= \alpha_t(y^{\hat{s}}, \hat{s}) + \beta(y^{\hat{s}}, \hat{s}) f(y^{\hat{s}}, \mathcal{K}(\hat{s})) \\ &= \omega_t + h_t(y^{\hat{s}}, \mathcal{K}(\hat{s})) - U_t\end{aligned}$$

Then since $\mathcal{K}(s) = \mathcal{K}(\hat{s})$, we know that $h_t(y^s, \mathcal{K}(s)) = h_t(y^{\hat{s}}, \mathcal{K}(\hat{s}))$ whenever $y^s = y^{\hat{s}}$. Therefore $\sigma_t^F(y^s, s) = \sigma_t^F(y^{\hat{s}}, s)$ whenever $y^s = y^{\hat{s}}$. This and the above result that the core state jurisdiction cost shares are anonymous prove that the cost share system is fully anonymous. ■

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