Tiebout Economies with Differential Genetic Types and†
Endogenously Chosen Crowding Characteristics

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Abstract

We consider a Tiebout economy with differential crowding and public projects in which agents are distinguished by their tastes and genetic endowments. Agents choose which crowding characteristic, for example, skill, they wish to express, and this affects their value to other members of their jurisdiction, club, firm, etc. An agent's choice is influenced both by his genetic endowment, which affects his cost of acquiring crowding characteristics, and his preferences over which crowding characteristic he expresses. We show that if small groups are strictly effective, the core is equivalent to the set of anonymous competitive equilibrium outcomes, but that the core generally contains taste-homogeneous jurisdictions.

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1. Introduction

When public goods are subject to crowding, the benefits of forming large jurisdictions in order to share the costs of public goods are eventually offset by the negative externalities agents impose upon one another. Tiebout (1956) argued that this would give rise to a system of competing jurisdictions offering various combinations of public goods and taxes. These taxes, in turn, would form a kind of price system that would decentralize the optimal provision of public goods but would not depend on knowing the preferences of any specific agent. He further speculated that agents would find it in their best interests to segregate themselves by taste.

In this paper, we explore questions of anonymous decentralization and taste-homogeneity of optimal jurisdiction structures in the context of a general equilibrium Tiebout economy with crowding types. Our major innovation is to incorporate endogenous human capital accumulation into the model. We are able to show that in addition to the more traditional role of generating the efficient provision of public goods and allocating agents over jurisdictions, anonymous competitive prices can be used to induce agents to make optimal educational investment and labor market choices. We also show that when agents face *different* costs of acquiring externality generating crowding effects (for example, skills), then in general optimal jurisdictions are not taste-homogeneous. This contrasts with prior findings for simpler endogenous crowding types models in which agents face the *same* cost of acquiring crowding characteristics and in which taste-homogeneous jurisdictions are optimal.

As an aside, although we have framed our results in terms of a Tiebout economy it is worth mentioning that this kind of locational problem is only one possible interpretation. We could equally well translate our results to apply to other types of coalitions such as clubs, social groups, firms, and in general, any collection of agents who both can exclude others from joining their group and are affected by the characteristics of the other members of their coalition. In addition, while the presence of local public goods helps motivate our assumption of small group effectiveness, it is in no way essential to our formal arguments. Thus, standard games of pure coalition formation are special case of our model.

A key variable in all models of Tiebout economies is the form of crowding. In the simplest case, crowding is assumed to be anonymous.[1] For example, if you are waiting in a line, you are affected only by the total number of people ahead of you. A broader approach is to allow for the possibility that agents care about the characteristics of agents in their coalition as well as their numbers. For example you may care about how many smokers and nonsmokers are in a restaurant you are about to enter as well as the total number of patrons. While it is certainly true in many situations that crowding is not anonymous, early formalizations of differentiated crowding incorporated a significant restriction. Specifically, an agent's "type" was assumed to determine both his preferences and his external effects on others. There is no evident reason that these two very different aspects of an agent should be linked.

This observation prompted Conley and Wooders (1997) to propose that a formal distinction be made between the publicly observable crowding characteristics of an agent and his unobservable tastes. While this *crowding types* approach broke this linkage, it also raised new questions. Specifically, although there should be no objection to taking tastes as exogenous, it is not as clear that an agent's external effects should be treated as such. Obviously, certain crowding characteristics like gender, race, looks, physical handicaps, and so on, may be exogenous or prohibitively expensive to change. But restricting attention to such characteristics limits the application of the crowding types model to a very specific class of external effects.[2]

Many of the most important external effects agents generate are the result of choices they intentionally make in response to market signals. For example, doctors and plumbers confer different benefits on their communities; theorists affect departments of economics differently than macroeconomists; living with people who speak your language may be easier than living with people who do not; dressing in the latest

[1] Barham and Wooders (1998) provide a survey.

[2] Racism and sexism are examples of problems that can be naturally treated in an exogenous crowding types model.

fashions may help make your fellow citizens feel they are living in a sophisticated cosmopolitan city. In all of these cases, the different types of externalities generated by agents are the result of endogenous choices. After all, no one is born a macroeconomist (we hope); the only way to become one is to voluntarily go through a long period of specific training and study.

Endogenizing agents' crowding characteristics greatly extends the class of problems that can be treated by crowding types models. For example, if we interpret crowding types as "skills" or "professions," we can begin to address questions relating to human capital accumulation in the context of local public goods economies and economies with jurisdictions and firms. One immediate conclusion is that the fourth assumption suggested by Tiebout in his seminal paper (that agents earn only dividend income and so the labor market does not affect their choice of jurisdiction) is unnecessary. The Tiebout mechanism is more robust than even Tiebout himself may have suspected. We find, as Tiebout suggests, that it is possible to decentralize the provision of public goods and the allocation of agents over jurisdictions through a price/tax system that does not depend agents' preferences. In addition, we show that these same prices will also induce agents to efficiently choose to invest in education and thereby allocate themselves optimally over the set of possible professions.

We introduce two important considerations that influence agents' choice of professions. First, we allow agents to have different basic abilities, called their *genetic endowments.* These genetic endowments, in turn, affect the subjective cost to an agent of acquiring any given skill. Thus, an agent endowed with high intelligence might find it very easy to get a Ph.D. in economics but very costly to sit through MBA classes. A less intelligent agent might see the relative costs as being exactly the opposite. Second, we assume agents have preferences over their own choice of profession. Some agents may like to be doctors while others may prefer to be lumberjacks. In general, we would not expect an agent's preferences to be related to his genetic endowments. There may be people who are good at being economics professors but who would really rather be rock stars, for example.

Formally, we assume that agents are described by their preferences and genetic endowments. These are uncorrelated and not publicly observable. The skills or crowding characteristics that agents choose to acquire, however, are publicly observable and are the only features of agents that directly affect the welfare of others. Agents face an "educational cost function" which specifies the cost of acquiring a given skill to an agent with a given genetic endowment. Independent of this cost, agents have preferences over which skill they acquire. An agent's chosen crowding type is therefore an argument in his preference mapping. Thus, an agent's choice of profession depends on the wages that various professions receive in equilibrium, how easy it is for him to join these professions given his genetic endowment, and how much he likes being a member of any given profession. We explore the effects of these features in the context of a general nontransferable utility economy, which includes as special cases the exogenous and endogenous crowding types, differentiated crowding, and anonymous crowding models. In particular, most of the prior Conley-Wooders results are subsumed and extended by this paper.

Our first result is that optimal jurisdictions may contain agents with different genetic and taste types. That genetic diversity within a coalition is beneficial is not very surprising. Such diversity allows jurisdictions to exploit the comparative advantages of different types of individuals. Somewhat more surprising is that taste-homogeneous jurisdictions are sometimes not optimal. We might expect that jurisdictions in which all agents agree on the most preferred bundle of public goods have an inherent advantage over jurisdictions in which agents have to compromise between competing views. Indeed, previous results (see Conley and Wooders 1996) for a simpler endogenous crowding types model suggested that this was the case. We show that the introduction of differential genetic types causes taste-homogeneity to break down.

Our second result is that the core can be decentralized by a set of admission prices that depend only on publicly-observable, endogenously-chosen crowding types, and not on unobservable, exogenously-given preferences or genetic endowments. In other words, the market will not permit discrimination on the basis of genetic endowments or preferences.[3] For example, the market does not respond to the difficulty an agent experiences in medical school or how much he enjoys his work. The market cares only that an agent be able to provide medical services of a specified quality. Thus, our model exposes a new role for competition between jurisdictions: the prevention of labor market biases on the basis of race, gender and other market-irrelevant aspects of an agent.

Our findings contrast with existing results for other treatments of crowding. Specifically, when crowding is anonymous, taste-homogeneity and anonymous decentralization both hold (Wooders 1978). When crowding is differentiated, however, both results fail (Berglas 1976 and Wooders 1985,1997). In an exogenous crowding types model, the core can be anonymously decentralized, but, surprisingly, jurisdictions in core states of the economy may not be taste homogeneous (Conley and Wooders 1995, 1997). On the other hand, when crowding types are endogenous, under equal educational access (that is, there are no genetic differences between agents), both results are again true (Conley and Wooders 1996). Finally, we show in this paper that when crowding types are endogenous and agents have different genetic endowments, the core can be anonymously decentralized but may not consist of taste homogenous jurisdictions. See Table 1 for a summary of these results.

Table 1 about here.

2. The Model

We consider an economy with one private good and a compact space $\mathcal{Y}$ of public

[3] This holds to the extent that genetic endowments do not preclude agents from acquiring any given crowding type. For example, Yoko Ono may be simply incapable of singing well, and therefore will never be paid like a Beatle. This conclusion is driven by our assumption that genetic endowments themselves have no crowding effects.

projects.[4] There are I agents indexed $i \in \{1, \dots, I\} \equiv \mathcal{I}$. Agents are distinguished by two factors: preferences and genetic endowments.

There are T different types of preferences, indexed by $t \in \{1, \dots, T\} \equiv \mathcal{T}$. The mapping $\tau : \mathcal{I} \to \mathcal{T}$ ascribes a taste type to each agent in the economy. Thus, if agent i is of taste type t, then $\tau(i) = t$.

Analogously, there are G different types of genetic endowments denoted $g \in \{1, \dots, G\} \equiv \mathcal{G}$. The mapping $\gamma : \mathcal{I} \to \mathcal{G}$ ascribes a genetic endowment to each agent in the economy. Both tastes and genetic endowments are assumed to be unobservable and are therefore private information.

Agents choose to acquire one of C different types of crowding characteristics, denoted $c \in \{1, \dots, C\} \equiv \mathcal{C}$. Crowding characteristics are assumed to be publicly observable; we can think of them as skills. An *assignment* is a mapping $A : \mathcal{I} \to \mathcal{C}$ which associates a choice of crowding type with each agent. Denote by $\mathcal{A}$ the set of all possible assignments.

We assume that agents with different genetic endowments may have different basic abilities. As a result, they may face different costs of acquiring a given crowding characteristic. This assumption is captured by the mapping

$$E : \mathcal{C} \times \mathcal{G} \to \Re,$$

called the *educational cost function.* Thus, the cost for an agent with genetic endowment g to become a crowding type c in terms of private good is $E(c, g)$. Note that educational costs are not restricted to be positive; some educational experiences may directly generate income.

The distinguishing feature of Tiebout economies is that agents find it optimal to break up into multiple jurisdictions which are small relative to the total population

[4] The use of public projects instead of Euclidian public goods follows a long tradition in the literature. This includes Littlechild (1975) for transferable utility games, Ellickson (1979) for public goods as indivisible commodities, and Mas-Colell (1980) in a general equilibrium context. Also see Manning (1992) and Demange (1994) for early treatments closer in spirt to the current paper. We should add that in the context of the model we develop below, generalizing to a compact space of projects comes at no cost and presents no issues of technical interest.

for the purposes of consuming public projects. An arbitrary jurisdiction of agents is denoted by $s \subset \mathcal{I}$, and $\mathcal{S}$ denotes the set of all possible jurisdictions. A list of jurisdictions $(s^1, \ldots, s^K) \equiv S$ is a *partition* if $\cup_k s^k = \mathcal{I}$ and, for all $s^k, s^{\bar{k}}$ such that $k \neq \bar{k}$, it holds that $s^k \cap s^{\bar{k}} = \emptyset$.[5]

As in differentiated crowding models, agents are affected (positively or negatively) by the particular mix of crowding characteristics possessed by the agents in the jurisdiction in which they reside. Denote a profile of crowding characteristics by $n = (n_1, \ldots, n_C) \in \mathbf{Z}_+^C$, where $\mathbf{Z}$ is the set of integers and n_c is interpreted as the number of agents in a jurisdiction who choose crowding type c. For any given assignment of agents to crowding types $A \in \mathcal{A}$, the *crowding profile* of a jurisdiction s is given by the mapping $CP : \mathcal{A} \times \mathcal{S} \rightarrow \mathbf{Z}_+^C$ defined by

$$CP(A, s) \equiv \left\{ n \in \mathbf{Z}_+^C \mid n_c = \mid s_c \mid \text{ where } i \in s_c \text{ if and only if } i \in s, \text{ and } A(i) = c \right\},$$

where $\mid \bullet \mid$ denotes the cardinality of a set.

A second factor that affects an agent's choice of crowding type is his own personal preferences over which characteristic he expresses. Thus, in addition to caring about the public projects and crowding profile of the jurisdiction he joins, an agent also cares about his own crowding type. Formally, each agent of taste type t has an endowment of private good $\omega_t \in \Re_+$ and a preference relation $\succeq_t$ defined over $\Re \times \mathcal{Y} \times \mathcal{C} \times \mathbf{Z}_+^C$. Denote a typical consumption bundle as (x, y, c, n), where x is a level of private good, y is a public project, c is the crowding type chosen by the agent and n is the crowding profile of the jurisdiction in which the agent resides.[6] We require only that the preference relations of agents satisfy monotonicity in the private good:

Monotonicity: For all $t \in \mathcal{T}$, $y \in \mathcal{Y}$, $c \in \mathcal{C}$, $n \in \mathbf{Z}_+^C$ and $x, \bar{x} \in \Re_+$ such that

[5] The script character ($\mathcal{S}$) denotes the set of all possible jurisdictions and the capital letter (S) denotes a partition.

[6] Since, by construction, only the crowding profile of an agent's jurisdiction affects his utility, we do not need to assume that preferences satisfy *taste anonymity in consumption* as we have in previous papers using the crowding types model. Similarly, we do not need to assume separately that production satisfies *taste anonymity in production*. Also note that, by construction, the genetic profile of a jurisdiction is irrelevant to either consumption or production. See Conley and Wooders (1995) for more details.

$x > \bar{x}$, $(x, y, c, n) \succ_t (\bar{x}, y, c, n)$.

Note that continuity and convexity are not required. Monotonicity certainly could be weakened to local nonsatiation, and could probably be dropped altogether, but at the cost of complicating the statements of results and their proofs.

Since an agent's genetic endowment is private information, no one but the agent himself knows the cost he incurs (that is, $E(A(i), \gamma(i))$) in acquiring a given skill. This information constraint renders it impossible to make an agent's jurisdiction directly responsible for his expenditure. Thus, we follow the convention that agents pay for their own education. There are two consequences. First, the private good level in a typical consumption bundle (x, y, c, n) must be interpreted as *gross private good consumption level.* Net of education costs, an agent only consumes $x - E(A(i), \gamma(i))$. This means, incidentally, that his gross consumption must be sufficient to pay for his education. Second, there are two reasons for an agent to care about his crowding type choice: (a) the direct effect of crowding-type choice on utility (e.g. an agent may enjoy being an artist more than a macroeconomist) and (b) the effect on net consumption of private good (e.g. to become a doctor is expensive and so an agent is worse off, all else equal, if this is his choice of profession). *Formally, both of these effects are accounted for by including an agent's crowding type choice in the consumption space over which preferences are defined.*

Crowding also affects production. The production technology, commonly available to all, is given by the cost function $f : \mathcal{Y} \times \mathbf{Z}_+^C \rightarrow \Re$ where

$$f(y, CP(A, s))$$

is the cost in terms of private good of carrying out a public project for jurisdiction s under assignment A.

A *feasible state of the economy,* (X, Y, A, S), is an allocation of private good for each agent, $X = (x_1, \ldots, x_I)$, a public project for each jurisdiction, $Y = (y^1, \ldots, y^K)$, an assignment A of agents to crowding types and a partition S of the population such that for all $i \in \mathcal{I}$,

$$x_i - E(A(i), \gamma(i)) \geq 0,$$

and

$$\sum_{i\in\mathcal{I}}(\omega_{\tau(i)} - x_i) - \sum_k f(y^k, CP(A, s^k)) \geq 0.$$

Denote the set of feasible states by F. The pair $(\bar{x}, \bar{y})$ is a *feasible allocation for a jurisdiction $\bar{s}$ under assignment $\bar{A}$* if, for all $i \in \bar{s}$,

$$\bar{x}_i - E(\bar{A}(i), \gamma(i)) \geq 0,$$

and

$$\sum_{i\in\bar{s}}(\omega_{\tau(i)} - \bar{x}_i) - f(\bar{y}, CP(\bar{A}, \bar{s})) \geq 0.$$

Note that we require that each agent's gross consumption of private good x_i is sufficient to pay for his education expenses $E(A(i), \gamma(i))$.

A jurisdiction $\bar{s} \in \mathcal{S}$ producing a feasible allocation $(\bar{x}, \bar{y})$ under assignment $\bar{A}$ *can improve upon* a feasible state $(X, Y, A, S) \in F$ if, for all $i \in \bar{s}$,

$$(\bar{x}_i, \bar{y}, \bar{A}(i), CP(\bar{A}, \bar{s})) \succeq_{\tau(i)} (x_i^k, y^k, A(i), CP(A, s^k)),$$

where $i \in s^k \in S$ in the original feasible state, and for some $j \in \bar{s}$ it holds that

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \succ_{\tau(j)} (x_j^k, y^k, A(j), CP(A, s^k)),$$

where $j \in s^k \in S$ in the original feasible state. A feasible state $(X, Y, A, S) \in F$ is in the *core* of the economy if it cannot be improved upon by any coalition.

We conclude this section with a discussion of the relationship of the crowding model described in this paper to previous approaches. By adding restrictions to this model we can obtain most of the previously described general equilibrium Tiebout models as special cases.[7] In particular, if there were only one genetic type and one costlessly acquired crowding characteristic, the model is equivalent to the anonymous crowding model introduced in Wooders (1978,1980a). Suppose instead that there were only one genetic type and there were identical numbers of taste and crowding types

[7] Exceptions include models with multiple private goods as in Wooders (1985,1997), for example.

(T=C). Suppose in addition that the cost of acquiring any crowding characteristic is zero and that, regardless of the allocation of public and private goods, an agent with any given taste type t always prefers to be crowding type c where $c = t$ rather than any other crowding type $\bar{c} \neq t$. Then, in equilibrium, agents of a given taste type will always choose the same crowding type. Thus, there is a perfect correlation between taste and crowding characteristics. This implies that the differentiated crowding model introduced in Wooders (1985) is also a special case.[8] Next, suppose there were the same number of crowding and genetic characteristics (C=G) and that it were free for type g's to acquire crowding type c, where $c = g$, and infinitely expensive to acquire any other crowding type. Then, in equilibrium there would be a perfect correlation between genetic and crowding characteristics. In effect, crowding type would become an exogenous characteristic. Thus, the exogenous crowding types model is a special case. Finally, if there were only one genetic type and agents were indifferent over their choice of crowding characteristic, the endogenous crowding type model with equal educational access becomes a special case.

It is important to note that endowing agents with genetic types instead of crowding types is not simply backing the endowments up by one level of notation. The introduction of genetic endowments and costs of acquiring crowding characteristics constitutes a substantial change both formally and economically from the differentiated crowding model and the crowding types model with exogenous types. The most important difference is that in the model described in this paper, the numbers of each crowding type are only determined in equilibrium. In contrast, in the exogenous crowding types case, if you start out with five lawyers, you end up with five lawyers. Labor supply is completely determined outside the model.

Introducing genetic types and endogenizing crowding characteristics also creates rich possibilities for addressing a variety of applied questions. Many of these questions cannot even be properly stated in an economy with exogenous crowding types. For ex-

[8] There are now a number of papers using this and the anonymous crowding model; see Barham and Wooders (1998) and Conley and Wooders (1998b) for further references.

ample, suppose that intelligence is a purely genetic characteristic, and that intelligence can be strictly ranked. Also suppose that more intelligent agents find every skill less expensive to learn than less intelligent agents, and that agents are indifferent over the which skill they acquire. A question we might ask is whether this implies agents form taste homogeneous jurisdictions. Our preliminary investigation shows the following: If the gap between the cost of acquiring each pair of crowding characteristics is the same across all genetic types, (for example, it might cost a smart type \$10 to be a lawyer and \$40 to be a doctor, while a dumber type might have to pay \$50 to be a lawyer and \$80 to be a doctor; thus, medicine costs the same \$30 cost premium over law for each type of agent) then the answer is yes. If instead one genetic type finds everything cheaper, but the cost differential between different crowding characteristics is not uniform across genetic types, then the answer is unclear. So far, we have been able to find neither a proof nor a counterexample. We also might wonder if highly intelligent agents would systematically choose different skills than less intelligent agents. The effects of directed educational subsides or technological changes in the production function on the equilibrium crowding mix or on the welfare of various types of agents could also be studied in such a model.

3. Equal Treatment and Strict Small Group Effectiveness

We now turn our attention to economies in which gains to coalition size are limited. Formally, an economy is said to satisfy *strict small group effectiveness*, (SSGE), if there exists a positive integer $B \in \mathbf{Z}_+$ such that

1. For all core states (X, Y, A, S) and for all $s^k \in S$ it holds that $\mid s^k \mid \leq B$.
2. If a feasible state (X, Y, A, S) be improved upon, then there exists a coalition $\bar{s} \in \mathcal{S}$ such that $\mid \bar{s} \mid \leq B$ which can also improve upon (X, Y, A, S).
3. for all $t \in \mathcal{T}$ and $g \in \mathcal{G}$, it holds that either $\mid \{i \in \mathcal{I} \mid \tau(i) = t, \gamma(i) = g\} \mid > B$ or $\mid \{i \in \mathcal{I} \mid \tau(i) = t, \gamma(i) = g\} \mid = 0$.

The first condition says that in all core states, agents are partitioned into "small" jurisdictions, that is, jurisdictions bounded in size. The second condition says that all possibilities to improve on any state are exhausted by small coalitions. The last condition says that the population of agents is large relative to the optimal jurisdictions in the sense that there if there are any representatives of a particular type of agent in an economy, then there are at least enough of them to fill up the largest potentially optimal jurisdiction. In other words, no type of agent that appears at all in the economy is scarce.[9] This is a relatively strong formalized version of the sixth assumption in Tiebout's original paper. Our view is that assuming that the economy is large relative to the optimal jurisdictions is more in the spirit of a definition of a Tiebout economy than a restriction on such an economy. Without some form of small group effectiveness or boundedness of per capita payoff, agents would find it optimal live together in the grand coalition. Strict small group effectiveness ensures that all outcomes in the core have the equal treatment property (Wooders 1983). Several forms of such conditions now appear in the literature; see Wooders (1994a,b) for treatment of the relationship between various forms of small group effectiveness, per capita boundedness, and strict small group effectiveness.

Our first Theorem states that SSGE implies that all agents of a given type are equally treated in the core. While the result follows from Wooders (1983, Theorem 3) for large games we include it here for completeness. Proofs of all propositions may be found in the appendix.

Theorem 1. *Let (X, Y, A, S) be a core state of an economy satisfying SSGE. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\tau(i) = \tau(\hat{i}) = t$ and $\gamma(i) = \gamma(\hat{i}) = g$, if $i \in s^k$, and $\hat{i} \in s^{\hat{k}}$ then*

$$(x_i, y^k, A(i), CP(A, s^k)) \sim_t (x_{\hat{i}}, y^{\hat{k}}, A(\hat{i}), CP(A, s^{\hat{k}})).$$

The last theorem of this section states that agents in jurisdictions with the same

[9] Note that we only require parts one and three of this definition for all of the results given in this paper except for Theorem 5 (which says that all core states can be decentralized though anonymous prices). We could dispense with part two of this definition altogether if we were willing to restrict the price system to include only jurisdictions with B or fewer agents.

crowding profile and public projects who choose to become the same crowding type must make the same implicit contribution to public goods production. This is the basic result that allows us to show the existence of anonymously decentralizing prices.

Theorem 2. *Let* (X, Y, A, S) *be a core state of an economy satisfying SSGE, and let* $s^k, s^{\hat{k}} \in S$ *be a pair of jurisdictions in the core partition such that* $y^k = y^{\hat{k}}$*, and* $CP(A, s^k) = CP(A, s^{\hat{k}})$*. Then for any crowding type* $c \in \mathcal{C}$*, and any pair of agents* $i \in s^k$ *and* $\hat{\imath} \in s^{\hat{k}}$ *such that* $A(i) = A(\hat{\imath}) = c$*, it holds that*

$$\omega_{\tau(i)} - x_i = \omega_{\tau(\hat{\imath})} - x_{\hat{\imath}}.$$

It is important to note that in a core state of the economy, an agent's implicit contribution to production of public projects ($\omega_{\tau(i)} - x_i$ in the above theorem) may be negative or positive. A negative contribution would mean in effect that an agent is being paid to be a member of a jurisdiction. This might be because he provides a very valuable skill to the society, for example. It might even be that his skill is sufficiently difficult to obtain that his price for jurisdiction membership embodies a net subsidy to make his education affordable.

4. Taste-Homogeneity and the Core

A central concern of this paper is whether taste-homogeneous jurisdictions are optimal. This has been an important issue in the Tiebout literature from its inception. If taste-homogeneity is in fact optimal, we can conclude a great deal about the efficient way to organize many types of social institutions. For example, it would suggest that homogeneous suburbs with no low income housing are socially preferred to the recently proposed scattered site housing plans, social organizations like the Lions club and country clubs should consist only of similarly minded people, it is better to educate students at specialized institutions like the London School of Economics, Caltech or the

Julliard School of Music than at more diverse universities, departments of economics would be more productive if all their members shared the same intellectual tastes and so on. It turns out that in general, taste-homogeneous jurisdictions are optimal when crowding is anonymous but not necessarily optimal when crowding is differentiated. Taste-homogeneous jurisdictions are also not necessarily optimal when crowding types are exogenously given. On the other hand, taste-homogeneous jurisdiction are optimal if crowding types are endogenous, but agents have the same genetic abilities. The main point of this section is that homogeneity breaks down when agents have either differential genetic endowments or preferences over the crowding type they choose. (See the conclusion for a more complete discussion of these results and of the existing literature.)

To state the idea of taste-homogeneity rigorously, we need to know which taste types are represented in a given jurisdiction. Let $\theta : \mathcal{S} \rightarrow \mathcal{T}$ be the function, giving a list of these types defined as:

$$\theta(s) \equiv \{t \in \mathcal{T} \mid \exists\ i \in s \text{ such that } \tau(i) = t\}.$$

Formally, a state (X, Y, A, S) is said to satisfy *strong essential taste-homogeneity* (SET) under the following conditions:

SET: Consider any jurisdiction in the core partition $s^k \in S$. Take any agent $i \in s^k$ and suppose that $\gamma(i) = g$ and $\tau(i) = t$. Then there exists a jurisdiction $\bar{s} \in \mathcal{S}$ and allocation $(\bar{x}, \bar{y})$ which is feasible for $\bar{s}$ under some assignment $\bar{A} \in \mathcal{A}$ such that $\theta(\bar{s}) = \{t\}$ and for all $j \in \bar{s}$ it holds that

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \succeq_{\tau(j)} (x_j, y^{\hat{k}}, A(j), CP(A, s^{\hat{k}}))$$

where, in the core state, $j \in s^{\hat{k}} \in S$.

In words, a state is strongly essentially taste-homogeneous if it is possible to take any jurisdiction s^k in the core partition which contains at least one agent of taste-type t and form a new jurisdiction containing only agents of type t while leaving all of its members at least as well off as they were in the core state. More succinctly, it is possible to "homogenize" any jurisdiction by taste without loss of utility to its members.[10]

Intuition arising from the existing literature strongly suggests that core states should satisfy SET. In a crowding types model there is no tension between the gains from trade motivation for mixing workers with different labor skills and the gains from specialization motivation for mixing only agents who share the same tastes for public goods. Thus, when taste-homogeneous jurisdictions containing a full array of genetic and crowding types are feasible, we would expect that such coalitions would do at least as well as taste-heterogeneous ones.[11]

It is somewhat surprising, therefore, that SSGE is not sufficient to imply that all core states satisfy SET, as the counterexamples below show. In the interest of transparency, these examples are in the form of simple matching problems.[12] The general proposition that strongly taste-homogeneous states may be strictly worse for agents than taste-heterogeneous states even under SSGE in no way depends on this simplification.

Example 1. Nonoptimality of taste-homogeneous jurisdictions when agents have different genetic types but are indifferent over their own crowding type.

Suppose agents choose to be one of two crowding types, Friendly and Unfriendly, F and U, respectively. Also suppose there are two taste types, Lovers and Haters of social interaction, L and H, respectively. Finally assume that there are two genetic types, Outgoing and Shy, O and S respectively. Let there be 100 agents of each all four possible taste and genetic types: $OL, OH, SL,$ and SH. In the interest of simplicity we

[10] We are implicitly assuming that the population of agents is sufficient to support the creation of taste-homogeneous jurisdictions. Otherwise, in a trivial sense, taste-homogeneity is not feasible, and therefore, not optimal. See Conley and Wooders (1996) for further discussion on this point.

[11] For an example of this intuition at work see Brueckner (1994) who explores a model related to crowding types in which taste-homogeneity depends on the relative strength of the preferences for public goods verses crowding externalities.

[12] Matching problems are, of course, a special case of the model presented in this paper. They are simpler in that agents receive utility only through the agent with whom they are partnered, and not explicitly through private or public consumption.

suppress the existence of public projects in this example. This allows us to use $U_t(\{\bullet\})$ to denote the utility received by any agent of type $t \in \mathcal{T}$ when living in a jurisdiction with a given mix of crowding characteristics. For example, $U_L(\{F, F\})$ is interpreted as the utility received by an agent who loves to socialize in a jurisdiction consisting of two agents who have chosen to be friendly types. The utility functions are given by the following:

$$U_H(\{F, F\}) = 0, \quad U_L(\{F, F\}) = 10,$$

$$U_H(\{F, U\}) = 5, \quad U_L(\{F, U\}) = 5,$$

$$U_H(\{U, U\}) = 10, \quad U_L(\{U, U\}) = 0,$$

and the utility received from being in every other possible type of jurisdiction is zero. In addition, let the educational cost function be as follows:

$$E(O, F) = E(S, U) = 0,$$

and

$$E(S, F) = E(O, U) = 100.$$

Under these conditions it always optimal for agents to use their comparative genetic advantages when choosing a crowding type. Thus, outgoing agents will always choose to be friendly types and shy agents will always choose to unfriendly types. This implies the following value function for the associated game:

$$V(\{FL, FL\}) = V(\{UH, UH\}) = 20,$$

$$V(\{FH, UL\}) = V(\{FL, FH\}) = V(\{FL, UL\}) = 10,$$

$$V(\{FL, UH\}) = V(\{FH, UH\}) = V(\{UL, UH\}) = 10,$$

$$V(\{FH, FH\}) = V(\{UL, UL\}) = 0,$$

and zero for every other jurisdiction type. By construction, the core will consist of jurisdictions with exactly two agents. Thus, since 100 agents of each type appear in the population, SSGE is satisfied, and fully taste-homogeneous jurisdictions are feasible. One core state consists of 50 jurisdictions each with compositions $\{OL, OL\}$, and $\{SH, SH\}$, and 100 jurisdictions with composition: $\{OH, SL\}$, with agents of type FL and UH receiving ten units of utility, and agents of type UL and FH receiving five units of utility. It is easy to check that this state cannot be improved upon.

We claim that it is not possible to taste-homogenize the mixed jurisdictions without loss of utility. Take a jurisdiction with composition $\{OH, SL\}$. Suppose we tried to taste-homogenize this jurisdiction by replacing the agent of type SL with one of type SH. This jurisdiction receives a total payoff of 10, while the sum of the core payoffs to these agents is 15. Thus, it is impossible to make these agents as well off in the taste-homogeneous jurisdiction as they are in the core state. It is easy to check that any other effort to taste-homogenize the mixed jurisdictions fails in the same way. We conclude that when agents have different genetic types but are indifferent over their own crowding type, the core will not in general satisfy strong essential taste-homogeneity.

$\blacksquare$

Example 2. Nonoptimality of taste-homogeneous jurisdictions when agents have the same genetic type, but care about their own crowding type.

Suppose agents choose to be one of two crowding types, Workers and Managers, W and M, respectively. Also suppose there are two taste types, people who like to work Indoors and people who like to work Outdoors, I and O, respectively. We assume that all agents are equally adept at acquiring either crowding characteristic. Equivalently, assume that all agents have the same genetic endowment. Let there be 200 agents, half of whom are "I's" and half of whom are 'O's". Again, we suppress public good production. To simplify notation, $U_t(c; n)$ will be used to denote the utility received by an agent of taste type $t \in \mathcal{T}$ when he chooses to be crowding type c and joins a jurisdiction with crowding profile n. To further simplify matters, we again assume that agents receive positive utility only when they are in jurisdictions containing exactly one worker and one manager. The utility functions are the following:

$$U_O(M; M, W) = 5, \quad U_I(M; M, W) = 10,$$

$$U_O(W; M, W) = 10, \quad U_I(W; M, W) = 5,$$

and the utility received from being in every other possible type of jurisdiction with any crowding type choice is zero. Finally let the educational cost function be:

$$E(W) = E(M) = 0.$$

Note that the education cost does not depend on genetic type since, by assumption, all agents have the same genetic endowment. Under these conditions outdoor agents should always choose to be workers and indoor agents should always choose to be managers. The core state always involves one outdoor type who has chosen to be a worker joining with one indoor type who has chosen to be a manager. This implies the following value function for the associated game:

$$V(\{O, I\}) = 20$$

$$V(\{O, O\}) = V(\{I, I\}) = 15$$

and zero for every other type of jurisdiction. One particular core allocation assigns each agent 10 units of utility. The important point is that there is a loss of utility from trying to match agents with the same taste type. We conclude that when agents have the same genetic type, but care about their own crowding type, in general the core will not be strongly taste-homogeneous.

■

The spirit of these two examples is the same. If agents are complementary either in the sense that they are good at generating a crowding externality desired by another type or they enjoy generating a crowding externality desired by another type, it may be optimal to mix agents across taste types.

Note that is easy to show the not-very-surprising result that in general it is also optimal for jurisdictions to be genetically heterogeneous.

While, in general, SET does not hold, a weak version of taste-homogeneity holds under relatively mild conditions. Specifically, SSGE alone implies that there is no advantage in mixing taste types *within a given crowding type.* That is, a jurisdiction can never do better by having a variety of taste types choosing to be a particular crowding type c than by having all agents who choose to be type c be of the same taste type. For example, this means that it is optimal for all doctors in a coalition to have the same tastes as each other and for all lawyers in a coalition to have the same tastes as each other. However, we would not expect doctors and lawyers in a given to coalition to share common tastes.

To state this idea rigorously, we need to know, for a given jurisdiction, the set of taste types of the individuals who have chosen a given crowding type $c \in \mathcal{C}$. The function $\theta_c : \mathcal{A} \times \mathcal{S} \to \mathcal{T}$ gives a list of these types:

$$\theta_c(A, s) \equiv \{t \in \mathcal{T} \mid \exists\ i \in s \text{ such that } \tau(i) = t \text{ and } A(i) = c\}.$$

Formally, a state (X, Y, A, S) is said to satisfy *weak essential taste-homogeneity* (WET) under the following conditions:

WET: Consider any $c \in \mathcal{C}$ and any jurisdiction in the core partition $s^k \in S$ such that $\mid \theta_c(A, s) \mid\ > 1$. Take any agent $i \in s^k$ such that $A(i) = c$, and suppose that $\tau(i) = t$. Then there exists a jurisdiction $\bar{s} \in \mathcal{S}$ and allocation $(\bar{x}, \bar{y})$ which is feasible for $\bar{s}$ under some assignment $\bar{A} \in \mathcal{A}$ such that $\theta_c(\bar{A}, \bar{s}) = \{t\}$ and for all $j \in \bar{s}$ it holds that

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \succeq_{\tau(j)} (x_j, y^{\hat{k}}, A(j), CP(A, s^{\hat{k}}))$$

where $j \in s^{\hat{k}} \in S$ in the core state.

In words, a core state is weakly essentially taste-homogeneous if the following is true. Choose any $c \in \mathcal{C}$ and take any jurisdiction s^k in the core partition which contains at least one agent of taste-type t choosing crowding type c. It is possible to form a new jurisdiction in which all agents who choose crowding type c are of taste type t and which leaves all of its members at least as well off as they were in the core state. More succinctly, it is possible to "homogenize" any jurisdiction by taste *within a given crowding type* without loss of utility.

Theorem 3. *If an economy satisfies SSGE, the core of the economy satisfies WET.*

5. Anonymous Decentralization and Core Equivalence

Tiebout's claim that when public goods are local, competitive forces cause the free rider problem to disappear, depends critically of the existence of a decentralizing price system that does not depend on agents' private information. In particular, price systems must not depend on agents' preferences or identity (as Lindahl prices typically do, for example) or on agents' genetic types. A price system satisfying this requirement is said to be *anonymous*. The main point of this section is to show the equivalence of the set of anonymous Tiebout admission price equilibrium states and the core. This equivalence has two important implications in the context of the current model. First, it shows that under SSGE, the market will prevent discrimination on the basis of nonexternality generating characteristics of agents. Even if it were possible to observe an agent's tastes and genetic endowment, any equilibrium price system which supports the core states would necessarily ignore this information. Second, to the list of objectives that Tiebout prices are able accomplish we can add the task of decentralizing optimal educational investment and labor market choices.

Following Conley and Wooders (1997), we take a price system for public projects as a mapping from a domain of crowding profiles and public projects into net private goods contributions. We need one natural restriction on the domain of this mapping: no admission price is defined for an agent who chooses to be crowding type c to join a jurisdiction that contains no agents of this type. For example, if a jurisdiction is constructed in such a way that it contains no lawyers, it is illogical to define an admission price to this jurisdiction for lawyers. Obviously, a lawyer cannot be a member a jurisdiction containing no lawyers. To define the price system, let $\mathcal{N}_c$ denote the set of crowding profiles which include at least one agent of type c:

$$\mathcal{N}_c \equiv \{n \in \mathbf{Z}_+^C \mid n_c > 0\}.$$

A price system for an agent who has chosen to be crowding type c is given by the mapping:

$$\rho_c : \mathcal{Y} \times \mathcal{N}_c \rightarrow \Re,$$

where $\rho_c(y, n)$ is the price that an agent who chooses to be crowding type c would have to pay to join a jurisdiction producing public good levels y and having a crowding profile n. Note that this price system is anonymous; it depends only on the observable characteristics of agents (crowding types) and not on unobservable characteristics (tastes and genetic endowments). A *Tiebout admission price system* denoted by ρ is simply a collection of such price systems (one for each crowding type).

A *Tiebout equilibrium* is a feasible state $(X, Y, A, S) \in F$ and a price system ρ such that:

1. For all $s^k \in S$, all individuals $i \in s^k$, all alternative crowding profiles $\bar{n} \in \mathbf{Z}_+^C$, all alternative crowding choices c such that $\bar{n}_c > 0$, and for all alternative public projects $\bar{y} \in \mathcal{Y}$,

$$(\omega_{\tau(i)} - \rho_{A(i)}(y^k, CP(A, s^k)), y^k, A(i), CP(A, s^k)) \succeq_{\tau(i)} (\omega_{\tau(i)} - \rho_c(\bar{y}, \bar{n}), \bar{y}, c, \bar{n}).$$

2. For all potential jurisdictional crowding profiles $\bar{n} \in \mathbf{Z}_+^C$ and public projects $\bar{y} \in \mathcal{Y}$,

$$\sum_{\{c \in \mathcal{C} \mid \bar{n}_c > 0\}} \bar{n}_c \rho_c(\bar{y}, \bar{n}) - f(\bar{y}, \bar{n}) \leq 0.$$

3. For all $s^k \in S$,

$$\sum_{i \in s^k} \rho_{A(i)}(y^k, CP(A, s^k)) - f(y^k, CP(A, s^k)) = 0.$$

Condition (1) states that all agents maximize utility over jurisdiction type, public goods levels and crowding assignments. Condition (2) requires that given the price system, no firm can make positive profits by entering the market and offering to provide any jurisdiction with any public project. Condition (3) requires that all equilibrium jurisdictions are able to cover their costs.[13]

It is worth spending a few words explaining and motivating this equilibrium concept. At the most basic level, Tiebout admission price equilibrium is very much like any other competitive equilibrium notion. Under the specified prices, agents maximize their preferences while firms maximize their profits. Most importantly, these optimizations are carried out under anonymous prices.[14] In this respect, the price system we define is distinguished from the personalized price system seen in Lindahl equilibrium and is consistent with Tiebout's program. The anonymity property implies that no personalized lump-sum transfers are implicit in the price system. There is one feature of these prices, however, which is a bit unusual. The Tiebout admission price system has no linearity properties.[15] It provides a separate lump-sum admission price for every possible crowding profile and for every possible public project. Since the commodity space consists of all possible pairs (y, n), an infinite set of prices may be required. In contrast, competitive equilibrium for private goods economies requires as many prices as goods and even Lindahl equilibrium requires a finite number of prices. In certain cases, the dimensionally of our price system can be reduced. For example, if crowding is anonymous and we consider a Euclidean space of public good goods produced under constant returns to scale, then a finite dimensional price system can decentralize core states of the economy (Wooders (1978)). Also, If we limit the number of possible public goods bundles and/or admissible jurisdictions, then the price system can

[13] Sergiu Hart has pointed out to us that condition (3) is implied by condition (2) and the definition of feasibility. We state condition (3) because we wish to emphasize that equilibrium jurisdictions make zero profit, and thus that jurisdiction formation is competitive.

[14] This feature of our price system contrasts with Wooders (1989), Scotchmer and Wooders (1988) and subsequent papers which define an agent's type to include both his taste and crowding characteristics. Thus, the price systems in these models depend on private information (tastes) and are therefore not anonymous.

[15] The nonlinearity of the price system, incidentally, is the reason that we need not require that preference or production sets be convex.

be correspondingly reduced. But, unless we allow prices to be defined to depend on preferences, in general a complete price system must have an infinite number of prices. (See Conley and Wooders (1998c) and Conley and Smith (1997), who provide some examples illustrating these points).

The next theorem states that all equilibrium states are also core states. Note that this does not depend on SSGE. An immediate corollary is a first welfare theorem for Tiebout equilibria.

Theorem 4. *If the state $(X, Y, A, S) \in F$ and the price system ρ constitute a Tiebout admission price equilibrium, then (X, Y, A, S) is in the core.*

Now we show the converse. Note that we add the fairly innocuous assumption that preferences are continuous in private good. This simplifies the proof, but is probably not critical to the theorem.

Theorem 5. *If an economy satisfies SSGE and for all $t \in \mathcal{T}$, the preference relation $\succeq_t$ is continuous in x, then for each state (X, Y, A, S) in the core, there exists a price system ρ such that ρ and (X, Y, A, S) constitute a Tiebout equilibrium.*

It is an immediate corollary of Theorems 4 and 5 that the core and set of equilibrium states of the economy are equivalent.

Theorem 6. *If an economy satisfies SSGE and all agents have continuous preferences in private good, then the core and set of equilibrium states are equivalent*

Proof/

Immediate from Theorems 4 and 5.

■

The reader might wonder how a core equivalence theorem is possible despite the population being finite. Intuitively, the assumption of SSGE implies that no jurisdiction in a core state can contain all agents of any one type. Given that it is possible to transferring well-being from one individual to another,[16] we conclude that all states of the economy in the core have the equal treatment property. Moreover, since SSGE implies all improvement can be carried out by coalitions bounded in size, the core does not shrink when the economy is replicated. Agents simply replicate the core state when they are added to the economy. Thus, the limiting equivalence of the core and the competitive allocations already holds for all sufficiently large finite economies. This is different from the case of pure public goods economies where there are increasing returns to group size and the core is large and may even grow as the population increases.

Equivalence of the core and equilibrium for finite economies is fairly robust. Provided that SSGE is satisfied, it holds for games as well as in economies with jurisdictions and/or local public goods. With multiple private goods, SSGE is not satisfied in a strict sense, but an asymptotic version of SSGE follows from the apparently mild assumption of boundedness of the supremum of per capita payoffs.[17] As a result, an asymptotic core convergence theorem is true for such economies. We discuss this type of "epsilon" result at more length below. Core equivalence results can also be obtained when more restrictions (finiteness or linearly, for example) are place on the equilibrium concept if the economic domain is also restricted (for example, to economies with a finite number of public projects or to convex economies).

6. Conclusion

Tiebout's (1956) most famous insight is that when public goods are provided to agents by competing jurisdictions, the market failure associated with a pure public

[16] For example, when there is a desirable, infinitely divisible commodity which is held in positive amounts by agents in any core state of the economy, or when there are quasi-linear utilities.

[17] cf. Wooders (1980b,1994a) provides convergence results for games satisfying boundedness of per capita payoffs and Wooders (1985,1997) demonstrates similar results for economies with local public goods and differentiated crowding. Shubik and Wooders (1982) treat core convergence in situations where agents may belong to multiple clubs.

goods economy will disappear. To put this more formally, Tiebout claimed that under these conditions it would be possible to anonymously decentralize the efficient provision of public goods and the allocation of agents over jurisdictions. Tiebout also hypothesized that the optimal jurisdiction would consist of agents who share the same tastes.

The main point of this paper is to further investigate the truth of these two fundamental claims about local public goods. We make a distinction between an agent's tastes and the external crowding effects he has on others. We assume that agents are endowed with basic genetic aptitudes which affect the cost of acquiring any given crowding characteristic. Agents are also assumed to care about the type of crowding effects they acquire. We allow for differentiated crowding in both consumption and production in a one-private-good, nontransferable utility general equilibrium economy with public projects. We note that models of Tiebout economies with anonymous crowding, differentiated crowding, exogenous crowding types, and endogenous crowding types with equal educational access are special cases of the model we describe here.

Our conclusions are especially interesting if we interpret crowding characteristics as skills acquired through educational choices made by agents. The fact that decentralizing prices distinguish amongst agents only on the basis of expressed crowding characteristics and not on the basis of unobservable tastes or genetic types is, in effect, a non-discrimination result. Under small group effectiveness, it is impossible to treat agents differently because of their tastes or genetic characteristics. However, the potential optimality of taste-heterogeneous and genetically-heterogeneous jurisdictions suggests that there are often advantages to diversity. These results show that it is really not necessary to exclude labor market considerations in order to obtain optimal outcomes in a Tiebout economy. Contrary to what Tiebout himself seems to have thought, competition between jurisdictions for agents with various skills results in efficient decentralization of labor market and educational decisions as well as efficient public good provision.

The effects of educational choice in models with coalition formation have been explored by several other authors in a variety of frameworks. Notable recent contributions include Epple and Romano (1998) who investigate the homogeneity and equilibrium tax structure of endogenously formed schools using simulation methods where student quality generates "peer-group effects." This work is continued in Epple, Newlon and Romano (1997), who study the effect of "tracking" students of various qualities within schools. In the context of a growth model, Benabou (1996) also considers questions of homogeneity and stratification of jurisdictions when parents decide where to live based in part on the level of educational funding and average student quality. Finally, see de Bartolome (1990) for a model with two crowding types (high and low ability students) which focuses on the characterization of equilibrium communities and land values when communities serve primarily as devices to provide education.

The major differences between these models and the model described in the current paper are twofold. First, the models discussed above are rich in detail[18] and are generally directed at addressing specific policy concerns. In contrast, our model is directed toward answering abstract questions in general equilibrium theory and imposes no structure on preferences, the form of crowding and so on. Second, in the models listed above, education has the flavor of a consumption good. We, on the other hand, treat education as a means to the ends of receiving higher income and achieving greater job satisfaction.

One issue that we do not treat at all is the question of whether equilibrium exists and the core is nonempty. There is a large literature that addresses this point in a variety of contexts going from Rose-Ackerman (1979) and Westhoff (1977) through Konishi (1997) and Nechyba (1997).[19] It is well known that in many cases the core and equilibrium will be empty in finite economies: see Bewley (1981) and Pauly (1970), for example. Our defense is the following. While exact equilibrium may not exist without

[18] For example, the form of crowding is restricted and there are a limited number of taste and crowding characteristics. In some cases, the number of jurisdictions is exogenously set and specific functional forms are used for utility functions as well.

[19] We apologize for any omissions.

special assumptions[20] various ϵ-equilibria exists and approximate cores are nonempty in quite general environments. To be very brief, an idea of ϵ-equilibrium is that there is a small per capita cost of setting up new jurisdictions. It turns out that this friction (which can be arbitrarily small for sufficiently large economies) is enough to regain existence.[21] Moreover, in a properly defined continuum limit, this friction can go to zero and an exact core is nonempty and is equivalent to set of equilibrium states. Conley and Wooders (1998a) demonstrate this result in the context of a crowding types model and provide a short survey of the existence literature. We chose not to state the results of this paper in these "ϵ" terms in interest of simplicity, but there is no technical factor that would prevent this if an existence result were desired.

In general, our findings suggest that the basic Tiebout hypothesis is fairly robust to different specifications of the model. We have focused on the addition of labor and human capital markets in this paper. Other authors have focused on such things as deciding public goods levels by voting (see, for example, Konishi 1996, and Page, Kollman and Miller 1997) or the introduction of multiple levels of public goods-providing governments (see Nechyba 1997). These results are also generally supportive of the Tiebout hypothesis. Yet other authors have taken the approach of replacing the core with various noncooperative equilibrium concepts like the Nash and coalition-proof Nash equilibrium. This program was initiated by Kalai, Postlewaite, and Roberts (1979) and has been continued more recently by Konishi, Le Breton, and Weber (1997). It seems in these cases that the efficiency of equilibrium holds only under fairly restrictive conditions. Which modeling strategy is most interesting or realistic is an open question. What is clear is that Tiebout economies provide a very rich field for research and have many important applications. They hold out real hope for a solution to the classic problem of free riding in the presence of public goods.

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[20] cf. Wooders (1978), Barham and Razzolini (1998).

[21] For this sort of result for economies with local public goods and anonymous crowding, see Wooders (1980a,1988). Such results also hold for large games (Wooders 1983 and, for some recent results and a survey, Wooders 1994a,b).

Appendix

Theorem 1. *Let (X, Y, A, S) be a core state of an economy satisfying SSGE. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\tau(i) = \tau(\hat{i}) = t$ and $\gamma(i) = \gamma(\hat{i}) = g$, if $i \in s^k$, and $\hat{i} \in s^{\hat{k}}$ then*

$$(x_i, y^k, A(i), CP(A, s^k)) \sim_t (x_{\hat{i}}, y^{\hat{k}}, A(\hat{i}), CP(A, s^{\hat{k}})).$$

Proof/

Suppose not. By SSGE, for all $s^k \in S$, it holds that $\mid s^k \mid \; \leq B$, and for this particular g, and t it holds that $\mid \{i \in \mathcal{I} \mid \tau(i) = t, \text{ and } \gamma(i) = g\} \mid \; > B$. It follows that there are least two individuals of type (g, t) who are in different jurisdictions in the core partition S. Since there exists at least one pair of individuals of type (g, t) who are not equally treated, we conclude that there must also be a pair of agents of this type are not equally treated and are in *different* jurisdictions in the core partition. Without loss of generality, let this pair be $i \in s^k$ and $\hat{i} \in s^{\hat{k}}$ where $s^k \neq s^{\hat{k}}$.

Now suppose without loss of generality that

$$(x_i, y^k, A(i), CP(A, s^k)) \succ_t (x_{\hat{i}}, y^{\hat{k}}, A(\hat{i}), CP(A, s^{\hat{k}})).$$

We claim this is not possible in a core state. To see this, consider the jurisdiction $\bar{s} \equiv \{s^k/i\} \cup \hat{i}$. Let the allocation for $\bar{s}$ be $(\bar{x}, \bar{y})$ where $\bar{y} = y^k$. Define $\bar{x}$ and $\bar{A}$ as follows: for agent $\hat{i}$, $\bar{x}_{\hat{i}} = x_i^k$, and $\bar{A}(\hat{i}) = A(i)$ and for all $j \in \bar{s}$ such that $j \neq \hat{i}$, $\bar{x}_j = x_j^k$, and $\bar{A}(j) = A(j)$. In words, the jurisdiction $\bar{s}$ is formed by replacing agent i with agent $\hat{i}$. The allocation $(\bar{x}, \bar{y})$ for $\bar{s}$ is identical to (x^k, y^k) except that agent $\hat{i}$ is given agent i's allocation of the private good, and takes agent i's crowding type assignment. Then, by construction, $CP(\bar{s}) = CP(s^k)$ and $\sum_{j \in s^k} \omega_{\tau(j)} = \sum_{j \in \bar{s}} \omega_{\tau(j)}$ it follows that $(\bar{x}, \bar{y})$ is feasible under assignment $\bar{A}$ for jurisdiction $\bar{s}$. Clearly, for all $j \in \bar{s}$ such that $j \neq \hat{i}$,

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \sim_{\tau(j)} (x_j, A(j), y^k CP(A, s^k)),$$

and for agent $\hat{i}$

$$(\bar{x}\ _i, \bar{y}, \bar{A}(\hat{i}), CP(\bar{A}, \bar{s})) \sim_t (x_i, y^k, A(i), CP(A, s^k)) \succ_t$$

$$(\hat{x}\ _i, y^{\hat{k}}, A(\hat{i}), CP(A, s^{\hat{k}})).$$

In words, all agents in $\bar{s}$ are at least as well off, and agent $\hat{i}$ is strictly better off. Since it is also immediate by construction that, for all $j \in \bar{s}$,

$$\bar{x}_j - E(\bar{A}(j), \gamma(j)) \geq 0,$$

this allocation improves upon (X, Y, A, S) which contradicts the hypothesis that it is a core state.

■

Theorem 2. *Let (X, Y, A, S) be a core state of an economy satisfying SSGE, and let $s^k, s^{\hat{k}} \in S$ be a pair of jurisdictions in the core partition such that $y^k = y^{\hat{k}}$, and $CP(A, s^k) = CP(A, s^{\hat{k}})$. Then for any crowding type $c \in \mathcal{C}$, and any pair of agents $i \in s^k$ and $\hat{i} \in s^{\hat{k}}$ such that $A(i) = A(\hat{i}) = c$, it holds that*

$$\omega_{\tau(i)} - x_i = \omega_{\tau(\hat{i})} - x_{\hat{i}}.$$

Proof/

Suppose not. Without loss of generality, assume

$$\omega_{\tau(i)} - x_i > \omega_{\tau(\hat{i})} - x_{\hat{i}},$$

$\gamma(i) = g$ and $\tau(i) = t$.

By SSGE, for all $s^k \in S$, it holds that $\mid s^k \mid \leq B$, and for this particular g, and t it holds that $\mid \{i \in \mathcal{I} \mid \tau(i) = t, \text{ and } \gamma(i) = g\} \mid > B$. it follows that there must be at least one agent of type (g, t) who is not in jurisdiction s^k. Call this agent $\bar{i} \in s^{\bar{k}} \neq s^k$.

Now construct a new jurisdiction $\bar{s}$ by replacing agent $\hat{i}$ with $\bar{i}$ in jurisdiction $s^{\hat{k}}$. Formally, consider the jurisdiction $\bar{s} \equiv \{s^k / \hat{i}\} \cup \bar{i}$. Define the allocation $(\bar{x}, \bar{y})$ and assignment $\bar{A}$ for $\bar{s}$ as follows: let $\bar{y} = y^k$, for all $j \in \bar{s}$ such that $j \neq \bar{i}$, let $\bar{A}(j) = A(j)$ and $\bar{x}_j = x_j$, for agent $\bar{i}$ let $\bar{A}(\bar{i}) = c$ and $\bar{x}_{\bar{i}}$ satisfy:

$$\omega_t - \bar{x}_{\bar{i}} = \omega_{\tau(\hat{i})} - x_{\hat{i}}.$$

To see this that this is a feasible plan for $\bar{s}$ note that when agent $\hat{i}$ is replaced by agent $\bar{i}$, the net collection of private goods for public projects production is the same for jurisdiction $\bar{s}$ as it was for s^k:

$$\sum_{j \in s^k} (\omega_{\tau(j)} - x_j) = \sum_{j \in \bar{s}} (\omega_{\tau(j)} - \bar{x}_j).$$

But since $CP(A, s^k) = CP(\bar{A}, \bar{s})$, this is sufficient to fund the public projects. In addition, since for all $j \in s$, $x_j - E(A(j), \gamma(j)) \geq 0$, it follows that for all $j \neq \bar{i}$, $\bar{x}_j - E(\bar{A}(j), \gamma(j)) \geq 0$. For agent $\bar{i}$ note that

$$\omega_t - \bar{x}_{\bar{i}} = \omega_{\tau(\hat{t})} - x_{\hat{i}} < \omega_t - x_i.$$

Thus, $\bar{x}_{\bar{i}} > x_i$, which implies that

$$\bar{x}_{\bar{i}} - E(\bar{A}(\bar{i}), \gamma(\bar{i})) = \bar{x}_{\bar{i}} - E(c, g) > x_i - E(c, g) = x_i - E(A(i), \gamma(i)) \geq 0.$$

Finally, it is easy to see that for all $j \in \bar{s}$ such that $j \neq \bar{i}$,

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \sim_{\tau(j)} (x_j, y^k, A(j), CP(A, s^k)).$$

Since agents i and $\bar{i}$ are both type (g,t), by Theorem 1 agents i and $\bar{i}$ must have been equally treated in the original core state. But, by construction, $\bar{x}_{\bar{i}} > \bar{x}_i = x_i$. Then by monotonicity,

$$(\bar{x}_{\bar{i}}, \bar{y}, \bar{A}(\bar{i}), CP(\bar{A}, \bar{s})) \succ_t$$

$$(x_i, y^k, A(i), CP(A, s^k)) \sim_t$$

$$(x_{\bar{i}}, y^{\bar{k}}, A(\bar{i}), CP(A, s^{\bar{k}})).$$

In words, all agents in $\bar{s}$ are at least as well off and agent $\bar{i}$ is strictly better off. This allocation improves upon (X, Y, A, S), which contradicts the hypothesis that it is a core state.

■

Theorem 3. *If an economy satisfies SSGE, the core of the economy satisfies WET.*

Proof/

Choose any crowding type $c \in \mathcal{C}$, any jurisdiction in the core partition $s^k \in S$ such that $\mid \theta_c(A, s^k) \mid\ > 1$, and any agent $\bar{i} \in s^k$ such that $A(\bar{i}) = c$, $\gamma(\bar{i}) = g$ and $\tau(\bar{i}) = t$. Recall that $(n_1^k, \ldots, n_C^k) \equiv CP(A, s^k)$. Let s_c be an arbitrary jurisdiction of n_c^k agents such that for all $j \in s_c$, $\gamma(j) = g$, $\tau(j) = t$, and if $j \in s^k$, then $A(j) = c$. By SSGE, there exist enough agents in the population to form jurisdiction s_c. Also define the following jurisdiction: $s_{\bar{c}} = \{i \in s^k \mid A(i) \neq c\}$. Note that the two jurisdictions s_c and $s_{\bar{c}}$ are disjoint by construction. Let $\bar{s}$ be the union of these two jurisdictions:

$$\bar{s} = s_c \cup s_{\bar{c}}.$$

In words, $\bar{s}$ consists of the union of two groups. The first consists of all the agents in s^k who did not choose to be crowding type c under assignment A. The second is constructed as follows. Suppose there is at least one agent of type (g,t) in s^k who chose to be crowding type c under A. Then collect a group of n_c^k agents of type (g,t) from either outside coalition s^k or from the set of agents in s^k who already choose to be crowding type c under A.

Let the assignment $\bar{A}$ be defined as follows:

$$\bar{A}(i) = \begin{cases} A(i) & \text{if } i \in s_{\bar{c}} \\ c & \text{if } i \in s_c \end{cases}.$$

Note that by construction $\theta_c(\bar{A}, \bar{s}) = t \in \theta_c(A, s^k)$, and $CP(A, s^k) = CP(\bar{A}, \bar{s})$. Now construct the allocation $(\bar{x}, \bar{y})$ for $\bar{s}$ as follows:

(1) let $\bar{y} = y^k$;
(2) for all $j \in s_{\bar{c}}$ let $\bar{x}_j = x_j$;
(3) for all $j \in s_c$ let $\bar{x}_j = x_{\bar{i}}$.

We first show that this is feasible for $\bar{s}$. By construction,

$$\sum_{i \in s_{\bar{c}}} (\omega_{\tau(i)} - \bar{x}_i) = \sum_{\{i \in s^k \mid A(i) \neq c\}} (\omega_{\tau(i)} - x_i).$$

By Theorem 2, for all $i, j \in s^k$, such that $A(i) = A(j) = c$,

$$\omega_{\tau(i)} - x_i = \omega_{\tau(j)} - x_j.$$

Thus,

$$\sum_{i \in s_c} (\omega_t - \bar{x}_i) = \sum_{\{i \in s^k | A(i) = c\}} (\omega_{\tau(i)} - x_i).$$

It follows that,

$$\sum_{i \in \bar{s}} (\omega_{\tau(i)} - \bar{x}_i) = \sum_{i \in s^k} (\omega_{\tau(i)} - x_i).$$

In addition, since $\omega_t - E(c, g) > 0$ and, for all $i \in s_{\bar{c}}$, $\bar{A}(i) = A(i)$, we know that for all $i \in \bar{s}$, it holds that $\omega_{\tau(i)} - E(A(i), \gamma(i)) > 0$. Therefore, $CP(A, s^k) = CP(\bar{A}, \bar{s})$ and $y^k = \bar{y}$, $(\bar{x}, \bar{y})$ is feasible for $\bar{s}$ under $\bar{A}$.

It only remains to show that all the agents in $\bar{s}$ are at least as well off as they were in the jurisdiction from which they came. This is immediate for $j \in s_{\bar{c}}$ since they receive exactly the same public projects and private goods level as they did in jurisdiction s^k. Now consider agents $i \in s_c$. Since, by construction, $\gamma(i) = \gamma(\bar{i}) = g$ and $\tau(i) = \tau(\bar{i}) = t$, by Theorem 1,

$$(x_i, y^{k_i}, A(i), CP(A, s^{k_i})) \sim_t (x_{\bar{i}}, y^k, c, CP(A, s^k)),$$

where $i \in s^{k_i} \in S$ in the core partition. Then since $\bar{x}_i = x_{\bar{i}}$, $\bar{y} = y^k$ and $CP(\bar{A}, \bar{s}) = CP(A, s^k)$, we conclude that for all $j \in \bar{s}$,

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \sim_{\tau(j)} (x_j, y^{k_j}, A(j), CP(A, s^{k_j})),$$

where $j \in s^{k_j} \in S$ in the core state.

■

Theorem 4. *If the state $(X, Y, A, S) \in F$ and the price system ρ constitute a Tiebout admission price equilibrium, then (X, Y, A, S) is in the core.*

Proof/

Suppose not. Then the Tiebout equilibrium state can be improved upon by some jurisdiction $\bar{s} \in \mathcal{S}$, providing an allocation $(\bar{x}, \bar{y})$ which is feasible under some assignment $\bar{A} \in \mathcal{A}$. Consider an arbitrary agent $i \in \bar{s}$ and suppose that he contributes more to public projects production in the improving jurisdiction than he would have been required to contribute under the price system:

$$\omega_{\tau(i)} - \bar{x}_i > \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s}))$$

But by definition of a Tiebout equilibrium,

$$(\omega_{\tau(i)} - \rho_{A(i)}(y^{k_i}, CP(A, s^{k_i})), y^{k_i}, A(i), CP(A, s^{k_i})) \succeq_{\tau(i)},$$

$$(\omega_{\tau(i)}\rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})), \bar{y}, \bar{A}(i), CP(\bar{A}, \bar{s}))$$

where $i \in s^{k_i} \in S$ in the equilibrium state. Then by monotonicity,

$$(\omega_{\tau(i)} - \rho_{A(i)}(y^{k_i}, CP(A, s^{k_i})), y^{k_i}, A(i), CP(A, s^{k_i})) \succ_{\tau(i)}$$

$$(\bar{x}_i, \bar{y}, \bar{A}(i), CP(\bar{A}, \bar{s})),$$

which contradicts the hypothesis that $(\bar{x}, \bar{y})$ is an improving allocation. Therefore, for all $i \in \bar{s}$,

$$\omega_{\tau(i)} - \bar{x}_i \leq \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})).$$

Observe that if $\sum_{i\in\bar{s}}(\omega_{\tau(i)} - \bar{x}_i) > f(\bar{y}, CP(\bar{A}, \bar{s}))$ then this jurisdiction can also improve on $(X, Y, A, S,)$ with another feasible allocation in which the surplus private good is redistributed back to the agents in $\bar{s}$. Thus, without loss of generality we may assume that

$$\sum_{i\in\bar{s}}(\omega_{\tau(i)} - \bar{x}_i) = f(\bar{y}, CP(\bar{A}, \bar{s}))$$

Suppose now that for some $i \in \bar{s}$,

$$\omega_{\tau(i)} - \bar{x}_i < \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})).$$

This implies that for some other $j \in \bar{s}$,

$$\omega_{\tau(j)} - \bar{x}_j > \rho_{\bar{A}(j)}(\bar{y}, CP(\bar{A}, \bar{s})).$$

This, however, contradicts what we show above. We conclude that for all $i \in \bar{s}$,

$$\omega_{\tau(i)} - \bar{x}_i = \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})).$$

By hypothesis, the allocation $(\bar{x}, \bar{y})$ and assignment $\bar{A}$ is improving for jurisdiction $\bar{s}$. This implies that for some $j \in \bar{s}$,

$$(\omega_{\tau(j)} - \rho_{\bar{A}(j)}(\bar{y}, CP(\bar{A}, \bar{s})), \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \sim_{\tau(j)},$$

$$(\bar{x}_j, \bar{y}, \bar{A}(j), CP(\bar{A}, \bar{s})) \succ_{\tau(j)}$$

$$(\omega_{\tau(j)} - \rho_{A(j)}(y^k, CP(A, s^{k_j})), y^{k_j}, A(j), CP(A, s^{k_j}))$$

where $j \in k_j \in S$ in the equilibrium state. However, this violates the definition of Tiebout equilibrium.

■

Theorem 5. *If an economy satisfies SSGE and for all $t \in \mathcal{T}$, the preference relation $\succeq_t$ is continuous in x, then for each state (X, Y, A, S) in the core, there exists a price system ρ such that ρ and (X, Y, A, S) form a Tiebout equilibrium.*

Proof/

By Theorem 2, for any $s^k, s^{\hat{k}} \in S$ such that (a) $y^k = y^{\hat{k}}$ and (b) $CP(A, s^k) = CP(A, s^{\hat{k}})$, for any crowding type $c \in \mathcal{C}$, and pair of agents $i \in s^k$, and $\hat{\imath} \in s^{\hat{k}}$ such that $A(i) = A(\hat{\imath}) = c$, it holds that

$$\omega_{\tau(i)} - x_i = \omega_{\tau(\hat{\imath})} - x_{\hat{\imath}}.$$

Therefore, it is consistent to define the admission price for pairs (y^k, s^k), which appear in the core state, as follows:

$$\rho_c(y, n) = \{\omega_{\tau(i)} - x_i \text{ where } A(i) = c, i \in s^{k_i} \in S, CP(A, s^{k_i}) = n, \text{ and } y^{k_i} = y\},$$

since for all such agents i, $\omega_{\tau(i)} - x_i$ is the same.

We claim that since (X, Y, A, S) is a core state, $(\{x_i\}_{i \in s^k}, y^k)$ must be a feasible allocation for jurisdiction s^k under assignment A, and that the private good collected under the prices defined above are exactly enough to fund the public project. Formally:

$$\sum_{i \in s^k} \rho_{A(i)}(y^k, CP(A, s^k)) \equiv \sum_{i \in s^k} (\omega_{\tau(i)} - x_i) = f(y^k, CP(A, s^k)).$$

Suppose instead that

$$\sum_{i \in s^k} (\omega_{\tau(i)} - x_i) > f(y^k, CP(A, s^k)).$$

Then jurisdiction s^k is making a net transfer of private good to the rest of the population, and therefore could improve on the core state by producing the same public project and distributing the private good surplus among its members. By the same argument, suppose

$$\sum_{i \in s^k} (\omega_{\tau(i)} - x_i) < f(y^k, CP(A, s^k)).$$

Then jurisdiction s^k is receiving a net transfer of private good from the rest of the population and therefore the rest of the population could improve on the core state by producing the same public project and distributing this transfer among its members. It follows that condition (2) of the definition of Tiebout equilibrium is satisfied by these prices.

It remains to be shown that it is possible to define admission prices for every other possible combination of public projects and crowding profiles such that agents are best off when they choose to remain in their core jurisdictions, and the profit from forming any such jurisdiction is nonpositive.

Consider an arbitrary $c \in \mathcal{C}$ and $(y, n) \in \Re^m \times \mathcal{N}_c$ which does not appear in the core state. Now take an arbitrary agent $i \in s^k \in S$ and consider how much he would be willing to pay to join this jurisdiction when he must choose to crowding type c. There are three possibilities. First, it may be that this jurisdiction is so attractive to agent i that even when he pays the most it is feasible for him to pay, (that is: $\omega_{\tau(i)} - E(c, \gamma(i))$)

he is still better off than he is in the core state. Second, it may be that this coalition is so unattractive that no amount of private good could make the agent as well off as he was in the core state. Third, it may be that there is an amount of private good which is feasible for the agent to pay, and which leaves him exactly indifferent between this coalition and the one he is in the core state. This creates a partition of agents which we define formally thus:[22]

$$Attr_c(y,n) \equiv \{i \in S \mid (E(c,\gamma(i)), y, c, n) \succ_{\tau(i)} (x_i, y^k, A(i), CP(A, s^k))\},$$

$$Unattr_c(y,n) \equiv \{i \in S \mid \forall \alpha \in \Re, (\alpha, y, c, n) \prec_{\tau(i)} (x_i, y^k, A(i), CP(A, s^k))\},$$

$$Indif_c(y,n) \equiv \{i \in S \mid \exists \alpha \in \Re \text{ s.t. } (\alpha, y, c, n) \sim_{\tau(i)} (x_i, y^k, A(i), CP(A, s^k))\}.$$

For agents in the set $Indif_c(y,n)$, the willingness to pay to join this jurisdiction is well defined. Define the maximum willingness to pay over all such agents as follows:

$$MaxWTP_c(y,n) = \max_{i \in Indif_c(y,m)} \{\alpha \in \Re \mid (\alpha, y, c, n) \sim_{\tau(i)} (x_i, y^k, A(i), CP(A, s^k))\}$$

Agents in the set $Attr_c(y,n)$ find that when they make their highest feasible contribution they are still better off than they were in the core. Define this maximum contribution as

$$MaxCont_c(y,n) = \max_{i \in Attr_c(y,n)} (\omega_{\tau(i)} - E(c,\gamma(i))$$

To complete the price system, choose $\epsilon > 0$ and let the admission price for any jurisdiction offering the public project and crowding profile (y,n), where for all k is the case that $y \neq y^k$, and $n \neq CP(A, s^k)$ (that is, (y,n) does not appear in the core state) as follows:

$$\rho_c(y,n) = \begin{cases} 1.\ \frac{-1}{\epsilon} & \text{if } Unattr_c(y,n) = \mathcal{I} \\ 2.\ MaxCont_c(y,n) + \epsilon & \text{if } Attr_c(y,n) \neq \emptyset \text{ and } Indif_c(y,n) = \emptyset \\ 3.\ MaxCont_c(y,n) + \epsilon & \text{if } Attr_c(y,n) \neq \emptyset, Indif_c(y,n) \neq \emptyset \\ & \text{and } Maxcont_c(y,n) \geq MaxWTP_c(y,n) \\ 4.\ MaxWTP_c(y,n) & \text{if } Attr_c(y,n) = \emptyset \text{ and } Indif_c(y,n) \neq \emptyset \\ 5.\ MaxWTP_c(y,n) & \text{if } Attr_c(y,n) \neq \emptyset, Indif_c(y,n) \neq \emptyset \\ & \text{and } Maxcont_c(y,n) < MaxWTP_c(y,n) \end{cases} .$$

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[22] The reader may wish to compare this to partition of agents in Diamantaras, Gilles, and Scotchmer (1996).

Suppose that case (1) obtains. Observe that no amount of private good is sufficient to induce any agent of type c to leave his current jurisdiction. Therefore, for all $\epsilon > 0$, all agents of type c are strictly worse off if they join a jurisdiction offering (y, n) at price $\rho_c(y, n)$. Suppose that case (2) or (3) obtains. For agents in $Attr_c(y, n)$ the price each agent is asked to pay to join the new jurisdiction exceeds their resources by ϵ and so (y, n) is not a feasible choice. Also, by construction, $\rho_c(y, n) = MaxCont_c(y, n) + \epsilon > MaxWTP_c(y, n)$ in this case, and so any agents in $Indif_c(y, n)$ are necessarily better off in their core jurisdictions. Finally, suppose that case (4) or (5) obtains. By construction, agents in $Indif_c(y, n)$ are exactly as well off in the new jurisdiction under these prices as they are in their core jurisdictions. Also, by construction, $\rho_c(y, n) = MaxWTP_c(y, n) > MaxCont_c(y, n)$, and so this jurisdiction is unaffordable to any agents in $Attr_c(y, n)$. Thus, condition one of the definition of the Tiebout equilibrium is satisfied.

It only remains to show that for any (y, n) where for all k is the case that $y \neq y^k$, and $n \neq CP(A, s^k)$ there exists $\epsilon > 0$ such that profits are nonpositive. First note that if for any $c \in \mathcal{C}$ case (1) holds, we can choose ϵ arbitrarily close to zero which makes the admission price for type c an arbitrarily large negative number. Obviously then, for small enough ϵ

$$\sum_{\{c \in \mathcal{C} | n_c > 0\}} n_c \rho_c(y, n) \leq f(y, n).$$

Now suppose that for at least one crowding type $c \in \mathcal{C}$ either case (2) or (3) holds, but for no crowding type does case (1) hold. Also suppose that for all $\epsilon > 0$

$$\sum_{\{c \in \mathcal{C} | n_c > 0\}} n_c \rho_c(y, n) > f(y, n).$$

Then for $\epsilon = 0$,

$$\sum_{\{c \in \mathcal{C} | n_c > 0\}} n_c \rho_c(y, n) \geq f(y, n).$$

But then (a) when agents pay these admission prices, there is enough private good to pay for producing the public project y, (b) there exist agents in $Attr_c(y, n)$ who are strictly better off when they join this coalition at these prices and for whom these prices are affordable and, (c) for crowding characteristics $c \in \mathcal{C}$ for which cases (4), or (5) hold, there exist agents in $Indif_c(y, n)$ who are exactly as well off joining this jurisdiction under these prices as they were in the core state. Thus, there exists a coalition for which (y, n) is a feasible and improving allocation. Since by SSGE part (2) we can assume without loss of generality that this improving coalition contains B or fewer agents, we conclude by part (3) of SSGE that their are enough agents in the population to form this coalition.[23] This contradicts the hypothesis that (X, Y, A, S) is a core state.

[23] This is the only point where we use part (2), which we can think of as exhaustion of blocking opportunities by small groups, in the paper.

Finally, suppose that every crowding type $c \in \mathcal{C}$ either case (4) or (5) holds. Also suppose

$$\sum_{\{c \in \mathcal{C} | n_c > 0\}} n_c \rho_c(y, n) > f(y, n).$$

By construction, all the agents who join the jurisdiction at these prices are exactly as well off as in the core state. Since the admission prices they pay more than cover the cost of the public project, an improving allocation is feasible by redistributing the surplus to the agents in this jurisdiction. This contradicts the hypothesis that (X, Y, A, S) is a core state. Therefore, condition three of the definition of the Tiebout equilibrium is satisfied at these prices.

■

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Type of Crowding	Anonymous Decentralization	Taste-Homogeneity
Anonymous Crowding	Yes	Yes
Differentiated Crowding	No	No
Exogenous Crowding Types	Yes	No
Endogenous Crowding Types with no Genetic Types	Yes	Yes
Endogenous Crowding Types with Genetic Types	Yes	No

Table 1. Anonymous Decentralization and Taste-homogeneity under Different Forms of Crowding