Taste-homogeneity of Optimal Jurisdictions in a Tiebout Economy†
with Crowding Types and Endogenous Educational Investment Choices

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Abstract

We examine a local public goods economy with differentiated crowding. The main innovation is that we assume that the crowding effects of agents are a result of choices that agents make. For example, agents may be crowded (positively or negatively) by the skills that other members of their jurisdiction possess and these skills may be acquired through utility maximizing educational investment choices made in response to equilibrium wages and educational costs. In such an environment, we show that taste-homogeneous jurisdictions are optimal. This contrasts with results for both for the standard differentiated crowding model, and the crowding types model. We also show that the core and equilibrium are equivalent, and that decentralization is possible through anonymous prices having a structure similar to cost-share equilibrium prices.

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Running head: Endogenous crowding types

1. Introduction

The key insight of Tiebout (1956) is that while people have an incentive to form groups in order to share the cost of public goods, this incentive might be limited by the negative external effects that agents impose on one another. In such an environment, agents break up into jurisdictions smaller than the entire population in order to efficiently provide themselves with public goods. He speculated that competition between these jurisdictions would induce agents to "vote with their feet" by choosing the jurisdiction that offered the best available mix of public goods and cost shares. Agents would thereby reveal their preferences, causing the free rider problem to disappear and the outcome to be efficient.

A second insight of Tiebout's classic paper is that placing agents with the different tastes in the same jurisdiction is likely to lead to conflict and inefficiency. There is a strong intuition that jurisdictions in which all agents agree on the most preferred mix of public goods can make their members better off than jurisdictions in which there is disagreement and agents are forced to reach a compromise. For example, a city in which half the citizens like parks and half like schools may only be able to provide half the levels of each of these two goods that is possible for cities of the same size containing citizens of only one type of citizen to offer. In economies in which the crowding is anonymous in the sense that agents are affected only by the numbers of agents with whom they share the collective goods, this intuition is essentially correct. Wooders (1978) shows that when small groups are effective,[1] all jurisdictions in core states of an economy will be demand-homogeneous. Demand-homogeneity means that all individuals in a jurisdiction will have the same *demands* for congestion and for public goods, although they may have different tastes.[2] While this is not the same as taste-homogeneity, it does imply that taste-heterogeneous jurisdictions can never Pareto

[1] See the definition in section two.

[2] See also Berglas and Pines (1980), Scotchmer and Wooders (1987) and Barham and Wooders (1996) for related results.

dominate those which are taste-homogeneous (although is some cases they may do exactly as well). We will say that such jurisdictions are *essentially taste-homogeneous.*

More generally, the external effects that one agent has on another may be positive or negative, may be felt through production or consumption, and in particular, may depend on an agent's type. When agents care not only about the number of agents in their jurisdiction but also about their types, crowding is said to *differentiated.* The optimality of homogeneous jurisdictions in differentiated crowding economies is much less clear. For example, if agents of type 1 are men, agents of type 2 are women, and the public good is a Saturday night dance, we would expect to see agents of both types mixing together in optimal jurisdictions (dance halls). It is not difficult to imagine any number of similar situations in which optimal jurisdiction are heterogeneous.

A reason for the optimality of heterogeneous jurisdictions is that there may be complementarities between the crowding effects of agents – the effects of agents on each other. In the standard differentiated crowding model an agent's type defines both his tastes and his crowding effects.[3] We have argued elsewhere (See Conley and Wooders 1995) that there is no reason to tie an agent's preferences to his external effects on others. One advantage to separating taste and crowding effects is that it allows us to define prices that depend only on observable crowding types. It is then possible to show that agents can be induced to sort themselves into an efficient pattern of jurisdictions in response to prices that are anonymous in the sense that they do not depend on agents' preferences. This extension of Tiebout's basic decentralization hypothesis cannot be obtained with the standard form of differentiated crowding since prices must depend on tastes in such models.

Another advantage to the crowding types model is that it allows us to study the optimality of taste-homogeneous jurisdictions when crowding is differentiated. For example, we can ask if Saturday night dances will be attended by people who all like the same types of music (have the same taste type) but by agents of both genders

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[3] See, for example, Berglas (1976) or Wooders (1981,1989, 1996) or Scotchmer (1994).

(of different crowding types). In the standard differentiated crowding model, it was impossible even to express such an question. The strong intuition from the anonymous crowding literature is that optimal jurisdictions in a crowding types model should be (essentially) taste-homogeneous. It is possible to take advantage of the full array of crowding effects without having to compromise with agents who have different tastes. It is surprising, therefore, that this turns out not to be true. In Conley and Wooders (1995) we show that even when small groups are effective and there are many agents of each type, it may still be that optimal jurisdictions are taste-heterogeneous.

Since the optimality of taste-heterogeneous jurisdictions is so counter-intuitive, it is natural to wonder if there are conditions under which taste-homogeneity can be recovered. We show in Conley and Wooders (1995) that the *hedonic independence* of agents' characteristics is one such sufficient condition. Hedonic independence means that the utility an agent receives in a core state can be explained solely as wages paid separately to his crowding and taste characteristics. In particular, there is no advantage or disadvantage to having any given combination of characteristics. Thus, the value of an agent's taste and crowding characteristics are independent. The problem with hedonic independence is that it is not generally satisfied in crowding types models. In fact, it may even be generically false. This leaves us without an explanation for why taste-homogeneous coalitions seem to be so common in everyday experience.

The major innovation in this paper is that we dispense with the assumption that crowding type is an exogenously given characteristic of agents. Exogeneity is appropriate when crowding type represents something like gender, height, intelligence or some other genetic endowment. It might also be acceptable when an agent's crowding type is a result of past and irreversible decisions, such as the choice to become a doctor or learn a language. On the other hand, it is clear that many characteristics which affect the welfare of others are the result of choices that agents make in response to market and other signals. For example, different types of economists provide different external benefits to their departments. Optimal departments have a mix of theorists, econometricians, macroeconomists and other specialties. Nobody, however, is born a macroeconomist (at least we hope not). Graduate students observe which fields are most in demand and so forth and then choose a specialty. This same type of decision making goes on at all levels of the labor and education markets, and even beyond. For example, people choose to get married, have children, become community leaders, own dogs, etc., at least partly in response to costs and benefits they can expect to receive as a result of providing their community the associated negative or positive external effects.

In this paper, we explore the effect of letting the crowding type of an agent be an endogenous characteristic which is chosen by agents in the context of a optimization model. To be concrete, we will discuss this in terms of agents facing a schedule of educational costs in order to acquire particular types of skills. There is nothing in the formal statement of the model, however, that excludes other types of decisions making. For example, married people may provide different types of externalities than single people, and it may cost something (search costs, perhaps) to become married.

We show that when agents choose their own crowding types, market forces make it impossible for any combination of tastes and crowding characteristics have utility generating value in excess of their independent market values. In other words, hedonic independence obtains as a result of agents' optimizing choice. This in turn implies that the core is taste-homogeneous. It also implies that there will exist anonymous decentralizing prices with a special, and very intuitive structure. Specifically, the price that an agent who chooses to acquire a given crowding type pays to join a jurisdiction with a certain profile of crowding characteristics and bundle of public goods will equal the average cost of providing the public goods plus the difference between the average education expenditures in this jurisdiction and his own educational expenditure.

2. The Model

We consider a one private good, M public goods economy with I agents indexed

$i \in \{1, \ldots, I\} \equiv \mathcal{I}$. Each agent can be one of T different types, indexed $t \in \{1, \ldots, T\}$. The mapping $\tau : \mathcal{I} \to \mathcal{T}$ is taken as the function which indicates the type of each agent in the economy. Thus, if agent i is of type t then $\tau(i) = t$.

An arbitrary coalition of agents is denoted $s \subset \mathcal{I}$, and $\mathcal{S}$ denotes the set of all possible coalitions. A list of coalitions $(s^1, \ldots, s^K) \equiv S$ is said to be a *partition* if $\cup_k s^k = \mathcal{I}$ and for all $s^k, s^{\bar{k}}$ such that $k \neq \bar{k}$, it holds that $s^k \cap s^{\bar{k}} = \emptyset$. Agents choose to acquire one of C different types of crowding characteristics denoted of $c \in \{1, \ldots, C\} \equiv \mathcal{C}$. These are assumed to be publicly observable and we can think them as skills. The cost to an agent of choosing of become any given crowding type is given by a mapping $E : \mathcal{C} \to \Re$ which is called the *educational cost function.* An *assignment* is a mapping $A : \mathcal{I} \to \mathcal{C}$ which associates with each agent a choice of crowding type. We denote by $\mathcal{A}$ the set of all possible assignments. An agent is affected (positively or negatively) by the particular mix of crowding characteristics possessed by the agents in his coalition. We denote a profile of crowding characteristics by $n = (n_1, \ldots n_C) \in R^C$ where n_c is an integer which is interpreted as the number of agents in a coalition with crowding type c. For any given assignment of agents to crowding types $A \in \mathcal{A}$, the *crowding profile* of a jurisdiction s is given by a mapping $CP : \mathcal{A} \times \mathcal{S} \to \Re^C$:

$$CP(A, s) \equiv \left\{ n \in \Re^C \mid n_c = \parallel s_c \parallel \text{ where } i \in s_c \text{ if and only if } A(i) = c \right\}$$

where $\parallel \bullet \parallel$ denotes the cardinally of a set.

Each agent of type t has an endowment of private good $\omega_t \in \Re_+$, and a quasilinear utility function $u_t : \Re \times \Re^m \times \Re^C \to \Re$. Thus, the utility an agent $i \in \mathcal{I}$ with tastes $t \in \mathcal{T}$ gets from consuming x private good and a vector y of public goods while in coalition s under assignment A is:[4]

$$u_t(x, y, A, s) = x - E(A(i)) + h_t(y, CP(A, s)),$$

[4] Since by construction only the crowding profile of an agent's coalition affects his utility, we do not need to assume that preferences satisfy *taste anonymity in consumption* as we have in previous papers using the crowing types model. Similarly, we do not need to assume separately that production satisfies *taste anonymity in production.* See, for example, Conley and Wooders (1995) for more detail.

Note we use the convention that each agent pays his own education costs. Thus, x is gross consumption which is divided between net consumption of private good and educational expenditures. We make no restrictions on the utility functions other than quasilinearity. In particular, continuity, convexity and monotonicity are not required.[5]

Crowding also affects production. The production technology, commonly available to all, is given by the cost function $f : \Re^m \times \Re^C \to \Re$ where

$$f(y, CP(A, s))$$

is interpreted as the cost in terms of private good of producing a vector y of public goods with coalition s under assignment A.

A *feasible state of the economy,* (X, Y, A, S), is a partition S of the population, an assignment A of agents to crowding types, an allocation $X = (x_1, \ldots, x_I)$ of private goods, and a set of public good production plans $Y = (y^1, \ldots y^K)$ such that

$$\sum_{i \in \mathcal{I}} (\omega_{\tau(i)} - x_i) - \sum_k f(y^k, CP(A, s^k)) \geq 0.$$

We denote the set of feasible states by F. We will also say that $(\bar{x}, \bar{y})$ is a *feasible allocation for a coalition* $\bar{s}$ *under assignment* $\bar{A}$ if

$$\sum_{i \in \bar{s}} (\omega_{\tau(i)} - \bar{x}_i) - f(\bar{y}, CP(\bar{A}, \bar{s})) \geq 0.$$

A coalition $\bar{s} \in \mathcal{S}$ producing a feasible allocation $(\bar{x}, \bar{y})$ under assignment $\bar{A}$ *can improve upon* a feasible state $(X, Y, A, S) \in F$ if, for all $i \in \bar{s}$:[6]

$$u_{\tau(i)}(\bar{x}_i, \bar{y}, \bar{A}, \bar{s}) > u_{\tau(i)}(x_i^k, y^k, A, s^k),$$

[5] The results we show this paper can extended to more general economies, for example, general ordinal preference and many private goods. We choose this simple framework to focus on the new results rather than technical generality.

[6] Since agents are assumed to have quasi-linear preferences, this is equivalent to requiring that no one agent be made better off, while keeping other agents at least as well off.

where $i \in s^k \in S$ in the original feasible state. A feasible state $(X, Y, A, S) \in F$ is in the *core* of the economy if it cannot be improved upon by any coalition.

3. Taste-Homogeneity and the Core

We now turn our attention to economies in which small groups are effective. An economy satisfies *strict small group effectiveness*, (SGE), if there exists a positive integer B such that:

1. For all core states (X, Y, A, S) and all $s^k \in S$, it holds that $\| s^k \| \leq B$.
2. for all $t \in \mathcal{T}$, it holds that $\| \{i \in \mathcal{I} \mid \tau(i) = t\} \| > B$.

The first condition says that any state which includes at least one jurisdiction with more than B agents can be improved upon. In other words, coalitions larger than B do strictly worse that coalitions with B agents or fewer. The second condition says that there are at least B agents of each type in the economy. This is a relatively strong formalized version of sixth assumption in Tiebout's original paper. Alternative definitions of strict small group effectiveness include assuming that all feasible utility vectors can be realized with partitions of the agents into jurisdictions containing no more than B members or that for sufficiently large replications of the economy, further replications do not increase per capita utilities. A less restrictive version, small group effectiveness, would require that groups bounded in size are able to achieve all or almost all per capita gains. More formally, given any epsilon greater than zero, there is an integer $B(\epsilon)$ such that groups can be constrained to be of size less then or equal to $B(\epsilon)$ with a loss due to this constraint of at most ϵ per capita. If sufficiently many agents of each type appear in the economy, this form of SGE is equivalent to the mild condition that per capita payoffs are bounded, introduced in Wooders (1979) to show nonemptiness of approximate cores.[7] Given this, our view is that the choice of form of

[7] See Wooders (1994b) for the relationship between SGE and per capita boundedness and Wooders (1994a) for the relationships between several forms of strict SGE and SGE.

SGE is largely a matter of convenience and so we choose a version that contributes to the simplicity of our proofs.

Our first theorem shows that SGE implies that all agents of a given type are equally treated in the core.

Theorem 1. *Let (X, Y, A, S) be a core state of an economy satisfying SGE. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\tau(i) = \tau(\hat{i}) = t$, if $i \in s^k$ and $\hat{i} \in s^{\hat{k}}$ then $u_t(x_i, y^k, A, s^k) = u_t(\hat{x}_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}})$.*

Proof/

Suppose not. By SGE, for all $s^k \in S$, it holds that $\parallel s^k \parallel \leq B$ and for all $t \in \mathcal{T}$, $\parallel \{i \in \mathcal{I} \mid \tau(i) = t\} \parallel > B$. Thus, if the theorem is false, it must be the case that there exists a pair of individuals $i, \hat{i} \in \mathcal{I}$, $t \in \mathcal{T}$ where $\tau(i) = \tau(\hat{i}) = t$, a pair coalitions s^k and $s^{\hat{k}}$, $s^k \neq s^{\hat{k}}$ where $i \in s^k$ and $\hat{i} \in s^{\hat{k}}$ such that

$$u_t(x_i, y^k, A, s^k) > u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}).$$

In words, there are at least two agents of the same type who are not equally treated and who are members of *different coalitions* in the partition S. We claim this is not possible in a core state. To see this, consider the coalition $\bar{s} \equiv \{s^k/i\} \cup \hat{i}$. Note that the allocation $(\bar{x}, \bar{y})$ where $\bar{y} = y^k$, for agent $\bar{i}$, $\bar{x}_{\hat{i}} = x_i^k$, and for all $j \in \bar{s}$ such that $j \neq \hat{i}$, $\bar{x}_j = x_j^k$, is feasible under assignment $\bar{A}$ where for agent $\bar{i}$, $\bar{A}(\hat{i}) = A(\hat{i})$ and for all $j \in \bar{s}$ such that $j \neq \hat{i}$, $\bar{A}(j) = A(j)$. In words, the coalition $\bar{s}$ is formed by replacing agent i with agent $\hat{i}$ and the allocation $(\bar{x}, \bar{y})$ of $\bar{s}$ is identical to (x^k, y^k) except that agent $\hat{i}$ is given agent i's allocation of private good, and takes agent i's crowding type assignment. This is feasible since by construction, $CP(\bar{s}) = CP(s^k)$. Note, therefore, that for all $j \in \bar{s}$ such that $j \neq \hat{i}$,

$$u_{\tau(j)}(\bar{x}_j, \bar{y}, \bar{A}, \bar{s}) = u_{\tau(j)}(x_j, y^k, A, s^k),$$

and for agent $\hat{i}$

$$u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}) = u_t(x_i, y^k, A, s^k) > u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}).$$

In words, all agents in $\bar{s}$ are at least as well off, and agent $\hat{i}$ is strictly better off. Then since all agents have quasilinear preferences is possible to redistribute some of agents $\hat{i}$'s gain to the other agents in $\bar{s}$ and leave all agents strictly better off. This improves upon (X, Y, A, S), which contradicts the hypothesis.

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Next we show that under small group effectiveness, the net contribution that each agents makes public goods productions (his endowment less his consumption of private good and educational expenditure) is the same of all agents in a given coalition regardless of the which crowding type an agent chooses. In other words, agents are compensated for their educational expenditure to the degree that all agents in a given coalition are indifferent over all possible choices of crowding type. Otherwise there would be an incentive for agents to choose different crowding type assignments.

Lemma 1. *For all core states (X, Y, A, S) of an economy satisfying SGE, for all $s^k \in S$ and for all $i, \hat{i} \in s^k$ it holds that $\omega_{\tau(i)} - x_i - E(A(i)) = \omega_{\tau(\hat{i})} - x_{\hat{i}} - E(A(\hat{i}))$.*

Proof/

Suppose not. Then without loss of with loss of generality, suppose

$$\omega_{\tau(i)} - x_i - E(A(i)) < \omega_{\tau(\hat{i})} - x_{\hat{i}} - E(A(\hat{i})).$$

By SGE there exists an agent $\bar{i} \in s^{\bar{k}}$ such that $s^{\bar{k}} \neq s^k$ and $\tau(\bar{i}) = \tau(\hat{i}) = t \in \mathcal{T}$. By Theorem 1 we know that

$$u_t(x_{\hat{i}}, y^k, A, s^k) = u_t(x_{\bar{i}}, y^{\bar{k}}, A, s^{\bar{k}}).$$

Let $\bar{A}$ an the assignment that is identical to A is all respects except that $\bar{A}(\bar{i}) = A(i)$. Consider the coalition $\bar{s} \equiv \{s^k/i\} \cup \bar{i}$ in which agent $\bar{i}$ replaces agent i in the coal ion s^k. Note that the allocation $(\bar{x}, \bar{y})$ where $\bar{y} = y^k$, $\bar{x}_{\bar{i}} = x_i^k$, and for all $j \in \bar{s}$ such that $j \neq \bar{i}$, $\bar{x}_j = x_j^k$, is feasible under assignment $\bar{A}$. By construction $CP(\bar{s}) = CP(s^k)$, and so for all $j \in \bar{s}$ such that $j \neq \hat{i}$

$$u_{\tau(j)}(\bar{x}_j, \bar{y}, \bar{A}, \bar{s}) = u_{\tau(j)}(x_j, y^k, A, s^k).$$

and

$$u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}) = u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}).$$

In addition,

$$f(\bar{y}, CP(\bar{A}, \bar{s})) = f(y^k, CP(A, s^k)).$$

But since

$$\omega_{\tau(i)} - x_i - E(A(i)) < \omega_{\tau(\hat{i})} - x_{\hat{i}} - E(A(\hat{i})),$$

it follows that

$$\sum_{j \in s^k} \left(\omega_{\tau(j)} - x_j - E(A(j))\right) < \sum_{j \in \bar{s}} \left(\omega_{\tau(j)} - x_j - E(A(j))\right).$$

By construction, the crowding profiles are same in these two coalitions, and as a result, so are the total educational expenditures. We conclude that

$$\sum_{j \in \bar{s}} \left(\omega_{\tau(j)} - x_j\right) - f(\bar{y}, CP(\bar{A}, \bar{s})) >$$

$$\sum_{j \in s^k} \left(\omega_{\tau(j)} - x_j\right) - f(y^k, CP(A, s^k)) \geq 0.$$

Thus, the coalition $\bar{s}$ collects has a surplus of private good after it pays for public goods productions. This surplus can redistributed to agents in $\bar{s}$ in a way that leaves them strictly better off. This improves upon (X, Y, A, S), which contradicts the hypothesis.

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We note as an aside that as Theorem 1 implies that each agent of type t gets the same utility level, U_t, this can be taken as a "wage" paid to the agents due to his taste-type. This in turn implies that no additional net "wage" is paid to an agents of a given type because of their choice of crowding type. We see in Lemma 1 that the difference net contribution to public goods productions between agents who choose different crowding types exactly offsets difference in educational costs. Thus, a very strong form of hedonic independence holds in this model. Since all crowding characteristics are equally available to all agents, competition between agents causes the net wage paid for choosing a particular crowding characteristic to be driven to zero. The utility each agent receives in the core is due solely to his tastes, and so is clearly independent of the crowding type he ends up choosing. This is in contrast to models in which crowding type is exogenous. The taste and crowding characteristics are not generally hedonically independent in this case. See Conley and Wooders (1996) for a formal definition of hedonic independence and further discussion.

Our main result in this section is that there is no advantage to mixing several types of agents in a single jurisdiction, although, as we point out in the introduction, it may the case that taste-heterogeneous coalitions do exactly as well as taste-homogeneous coalitions. Formally, a state (X, Y, A, S), is said to satisfy *strong essential taste-homogeneity* (SET) under the following conditions:

SET: for all taste types $t \in \mathcal{T}$ and every jurisdiction $s^k \in S$ in the core partition containing at least one agent i such that $\tau(i) = t$; and for every alternative jurisdiction $\bar{s} \in \mathcal{S}$ where $\tau(\bar{i}) = t$ for all $\bar{i} \in \bar{s}$; there exists an assignment $\bar{A} \in \mathcal{A}$ such that $CP(A, s^k) = CP(\bar{A}, \bar{s})$, and a feasible allocation $(\bar{x}, \bar{y})$ for $\bar{s}$ such that for all $\hat{i} \in \bar{s}$:

$$u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}) = u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}),$$

where $\hat{i} \in s^{\hat{k}} \in S$ in the original partition.

In words, a state is strongly essentially taste-homogeneous if when we choose any type $t \in \mathcal{T}$ that is represented in a given coalition s^k and form a new coalition $\bar{s}$ by replacing all agents not of type t with agents who are of this type, then choose crowding type assignments such that the crowding profile of the s^k is the same as the crowding profile of $\bar{s}$, it is possible to make all the agents in the $\bar{s}$ just as well off as they were in the original state. More succinctly, it is possible to "homogenize" any coalition along any set of tastes that are currently represented in the coalition without any loss of utility.

Theorem 2. *The core of an economy that satisfies SGE is strongly essentially taste-homologous*

Proof/

We must show for every core state (X, Y, A, S), that for all taste types $t \in \mathcal{T}$ and every jurisdiction $s^k \in S$ in the core partition containing at least one agent i such that $\tau(i) = t$; and for every alternative jurisdiction $\bar{s} \in \mathcal{S}$ where $\tau(\bar{i}) = t$ for all $\bar{i} \in \bar{s}$; there exists an assignment $\bar{A}$ such that $CP(A, s^k) = CP(\bar{A}, \bar{s})$ and a feasible allocation $(\bar{x}, \bar{y})$ for $\bar{s}$ such that for all $\hat{i} \in \bar{s}$:

$$u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}) = u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}),$$

where $\hat{i} \in s^{\hat{k}} \in S$ in the original partition.

Let $\bar{s}$ be a taste-homogeneous coalition that satisfies the conditions above, and let $\bar{A}$ be any assignment such that $CP(A, s^k) = CP(\bar{A}, \bar{s})$. By Lemma 1, all agents in a given coalition in a core state make the same net contribution to public goods production. Formally, we denote this net contribution as $z \equiv \omega_{\tau(j)} - x_j - E(A(j))$ for all $j \in s^k$. Consider the allocation $(\bar{x}, \bar{y})$ where $\bar{y} = y^k$, and for all $j \in \bar{s}$, $\bar{x}_j = \omega_t - z + E(\bar{A}(j))$. Note that by construction, coalition $\bar{s}$ and s^k have the same crowding profile, and produce the same level of public good, and collect the same total contributions from its members. It follows that $(\bar{x}, \bar{y})$ is feasible for $\bar{s}$ under assignment $\bar{A}$.

It only remains to show that all agents in $\bar{s}$ are at least well off with allocation $(\bar{x}, \bar{y})$ as they were in the original core state. Suppose instead that for some agent $\hat{i} \in \bar{s}$ such that in the original core state $\hat{i} \in s^{\hat{k}}$

$$u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}) > u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}).$$

Recall that by hypothesis, there exists an agent $i \in s^k$ such that $\tau(i) = t$, and so by Theorem 1,

$$u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}) = u_t(x_i, y^k, A, s^k).$$

But

$$u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}) = \omega_t - z + h_t(\bar{y}, CP(\bar{A}, \bar{s})) =$$

$$\omega_t - z + h_t(y^k, CP(A, s^k)) = u_t(x_i, y^k, A, s^k).$$

Thus,

$$u_t(x_{\hat{i}}, y^{\hat{k}}, A, s^{\hat{k}}) = u_t(\bar{x}_{\hat{i}}, \bar{y}, \bar{A}, \bar{s}),$$

a contradiction.

$\blacksquare$

4. Anonymous Decentralization and Core Equivalence

In Conley and Wooders (1995) we define a set of anonymous admission prices that allow the core states to be a decentralized. Such a price system gives an admission price that an agent of any given crowding type must pay to join coalitions with every possible crowding profile and for every possible public good level. Since we are only interested in decentralizing the core in this paper, and by SGE, the core consists only of coalitions of size B or smaller, we will restrict attention to systems that give prices only for coalitions that satisfy this requirement. Such restriction is not strictly necessary given the strength of the SGE assumption we make, however, it simplifies proofs. Formally:

$$N \equiv \{n \in \Re^C_+ \mid \sum_{c \in \mathcal{C}} n_c \leq B \text{ and } \forall\, c \in \mathcal{C}, n_c \text{ is an integer } \}.$$

In addition, it only makes sense to provide prices only for those coalitions that it is feasible for an agent who chooses a given crowding type to join. For example, if coalition with crowding profile n includes no agents of crowding type c ($n_c = 0$), then there is no reason to specify an admission prices for agents with crowding type c to this type of coalition.[8] More formally, let N_c denote set of vectors the nonnegative integers that sum to no more B:

$$N_c \equiv \{n \in N \mid n_c > 0\}.$$

A price system for crowding type c is given by a mapping:

[8] We emphasize that this does not mean that the price system excludes the possibility of coalitions with crowding profile n plus one agent of crowding type c. Such a coalition would have a crowding profile $\bar{n}$ where for all $\bar{c} \in \mathcal{C}$ such that $\bar{c} \neq c$, $n_{\bar{c}} = n_{\bar{c}}$, and $\bar{n}_c = 1$. Coalitions with crowding profile $\bar{n}$ would of course have an admission price defined for agents of crowding type c.

$$\rho_c : \Re^L_+ \times N_c \to \Re,$$

where $\rho_c(y, n)$ is interpreted as the price that an agent who chooses to be crowding type c would have to pay to join a coalition producing public good levels y and having a crowding profile n. Note that this price system is anonymous in the sense that it depends only of the observable characteristics of agents (crowding types) and not on unobservables (tastes).

A *Tiebout admission price system* is simply the collection of price systems for each crowding type and is denoted by ρ.

A *Tiebout equilibrium* is a feasible state $(X, Y, A, S) \in F$ and a price system ρ such that:

1. For all $s^k \in S$, all individuals $i \in s^k$, all alternative crowding profiles $\bar{n} \in N$, all alternative crowding assignments $\bar{A}$ and for all levels of public good production $\bar{y} \in \Re^L_+$,

$$\omega_{\tau(i)} - E(A(i)) - \rho_{A(i)}(y^k, CP(A, s^k)) + h_{\tau(i)}(y^k, CP(A, s^k)) \geq$$

$$\omega_{\tau(i)} - E(\bar{A}(i)) - \rho_{\bar{A}(i)}(\bar{y}, \bar{n}) + h_{\tau(i)}(\bar{y}, \bar{n}).$$

2. For all potential jurisdictions crowding profiles $\bar{n} \in N$ and public goods levels $\bar{y} \in \Re^L_+$,

$$\sum_{\{c \in \mathcal{C} | \bar{n}_c > 0\}} \bar{n}_c \rho_c(\bar{y}, \bar{n}) - f(\bar{y}, \bar{n}) \leq 0.$$

3. For all $s^k \in S$,

$$\sum_{i \in s^k} \rho_{A(i)}(y^k, CP(A, s)) - f(y^k, CP(A, s)) = 0.$$

Condition (1) says that all agents maximize utility over jurisdiction type, public goods levels and crowding assignments. Condition (2) requires that given the price system, no firm can make positive profits by entering the market and offering to provide any sort of jurisdiction. Condition (3) requires that all equilibrium jurisdictions make zero profit, and so cover their costs.[9]

The next theorem shows that all equilibrium states are also core states. An immediate corollary is that there is a first welfare theorem for Tiebout equilibrium

Theorem 3. *If the state $(X, Y, A, S) \in F$ and the price system ρ constitute a Tiebout admission price equilibrium, then (X, Y, A, S) is in the core.*

Proof/

Suppose not. Then the Tiebout equilibrium state can be improved upon by some jurisdiction $\bar{s} \in \mathcal{S}$, providing an allocation $(\bar{x}, \bar{y})$ which is feasible under assignment $\bar{A}$. Consider an arbitrary agent $i \in \bar{s}$, where $\tau(i) = t$. Suppose that in the Tiebout equilibrium state, i is a member of the jurisdiction $s^{k_i} \in S$. By definition, in the Tiebout equilibrium state, agent i's consumption of private good is

$$x_i \equiv \omega_t - \rho_{A(i)}(y^{k_i}, CP(A, s^{k_i})).$$

Suppose that jurisdiction $\bar{s}$ forms and agents pay admission prices given by the Tiebout pricing system instead of receiving the consumption levels they are assigned in the improving allocation. Denote these "Tiebout" consumption levels by

$$\tilde{x}_i \equiv \omega_t - \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})).$$

Since (X, Y, A, S) is a Tiebout state and by condition (1) of the definition of Tiebout equilibrium agents maximize utility under Tiebout prices, we know that:

$$\omega_t - E(A(i)) - \rho_{A(i)}(y^{k_i}, CP(A, s^{k_i})) + h_t(y^{k_i}, CP(A, s^{k_i})) \geq$$

$$\omega_t - E(\bar{A}(i)) - \rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A}, \bar{s})) + h_t(\bar{y}, CP(\bar{A}, \bar{s})).$$

Substituting and summing over all such agents $i \in \bar{s}$ yields

$$\sum_{i \in \bar{s}} \left(x_i - E(A(i)) + h_{\tau(i)}(y^{k_i}, CP(A, s^{k_i}))\right) \geq$$

[9] Sergiu Hart has pointed out to us that condition (3) is implied by condition (2) and the definition of feasibility. We continue to state condition (3) because we wish to emphasize that equilibrium jurisdictions make zero profit, and thus that club formation is competitive.

$$\sum_{i\in\bar{s}}\left(\tilde{x}_i - E(\bar{A}(i)) + h_{\tau(i)}(\bar{y}, CP(\bar{A},\bar{s}))\right).$$

But by the definition of improving coalition, for all $i \in \bar{s}$,

$$u_{\tau(i)}(\bar{x}_i, \bar{y}, \bar{A}, \bar{s}) > u_{\tau(i)}(x_i, y^{k_i}, A, s^{k_i}),$$

or equivalently,

$$\bar{x}_i - E(\bar{A}(i)) + h_{\tau(i)}(\bar{y}, CP(\bar{A},\bar{s})) > x_i - E(A(i)) + h_{\tau(i)}(y^{k_i}, CP(A, s^{k_i})).$$

Summing this over agents in $\bar{s}$ yields

$$\sum_{i\in\bar{s}}\left(\bar{x}_i - E(\bar{A}(i)) + h_{\tau(i)}(\bar{y}, CP(\bar{A},\bar{s}))\right) >$$

$$\sum_{i\in\bar{s}}\left(x_i - E(A(i)) + h_{\tau(i)}(y^{k_i}, CP(A, s^{k_i}))\right).$$

This implies that,

$$\sum_{i\in\bar{s}}\left(\bar{x}_i - E(\bar{A}(i)) + h_{\tau(i)}(\bar{y}, CP(\bar{A},\bar{s}))\right) >$$

$$\sum_{i\in\bar{s}}\left(\tilde{x}_i - E(\bar{A}(i)) + h_{\tau(i)}(\bar{y}, CP(\bar{A},\bar{s}))\right),$$

which allows us to conclude:

$$\sum_{i\in\bar{s}}\bar{x}_i > \sum_{i\in\bar{s}}\tilde{x}_i.$$

However, by the definition of improving jurisdictions, it holds that

$$\sum_{i\in\bar{s}}(\omega_{\tau(i)} - \bar{x}_i) - f(\bar{y}, CP(\bar{A},\bar{s})) \geq 0.$$

By the definition of a Tiebout equilibrium, it holds that

$$\sum_{i\in\bar{s}}\rho_{\bar{A}(i)}(\bar{y}, CP(\bar{A},\bar{s})) - f(\bar{y}, CP(\bar{A},\bar{s})) \equiv \sum_{i\in\bar{s}}(\omega_{\tau(i)} - \tilde{x}_i) - f(\bar{y}, CP(\bar{A},\bar{s})) \leq 0.$$

These two together imply

$$\sum_{i\in\bar{s}}\bar{x}_i \leq \sum_{i\in\bar{s}}\tilde{x}_i,$$

a contradiction

■

In Conley and Wooders (1995) we showed that all core states could be decentralized with anonymous admission prices. While it is of some interest to show that this result can be extend to a model with educational choice, the structure of the model allows us to say considerable more about the nature of prices. When crowding types are exogenous, we could only show the existence of decentralizing prices, but be could not say very much about their form. In general, they could be any (possibly nonlinear, nonconvex, or even discontinuous) function that mapped public goods levels and crowding profiles into the real numbers. We are able to show that the addition of endogenous choice of crowding types implies that prices depend in a very specific way on two factors. Specifically the next theorem demonstrates that following price system decentralizes the core:

$$\hat{\rho}_c(y, n) = \frac{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}} E(\bar{c})}{\sum_{\bar{c} \in \mathcal{C}} n_c} - E(c) + \frac{f(y, n)}{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}}}.$$

These prices say that all agents pay on equal share of the cost of public good, and in addition, only have to pay the average educational costs of the coalition they join. The first part is a special case of the cost share equilibrium. If it happens we have constant returns to scale, then these cost shares are equal linear cost shares, and given that the core is taste-homogeneous, these are Lindahl prices. This is an interesting contrast to Conley and Wooders (1995) in which we show that the possibility of such linear decentralization depends not only on the constant returns to scale of the production technology, but also on crowding being anonymous. It may be that making crowding type endogenous makes crowding anonymous in a Rawlsian “behind the veil” sense; before agents choose an assignment, they all have the same crowding potential. It is also interesting to compare this work to Weber and Weismeth (1992) who study equivalence of cost share equilibrium to the core in a pure public goods context. The main difference seems to be that we get equal cost shares due to the taste-homogeneity of optimal jurisdictions in our model, while in the Weber and Wiesmeth model with pure public goods, all agents of all types are in one jurisdiction and so the cost shares are not generally equal.

Theorem 4. *If an economy satisfies SGE, then for all states (X, Y, A, S) in the core $\hat{\rho}$ and (X, Y, n) form a Tiebout equilibrium.*

Proof/

First we show that profits are non-positive, and that and all equilibrium clubs cover costs under this price system. By definition,

$$\hat{\rho}_c(y,n) = \frac{\sum_{\bar{c}\in\mathcal{C}} n_{\bar{c}} E(\bar{c})}{\sum_{\bar{c}\in\mathcal{C}} n_c} - E(c) + \frac{f(y,n)}{\sum_{\bar{c}\in\mathcal{C}} n_{\bar{c}}}.$$

Thus, the total revenue provided by a jurisdiction with crowding profile n is:

$$\sum_{c\in\mathcal{C}} n_c E(c) - \sum_{c\in\mathcal{C}} n_c \frac{\sum_{\bar{c}\in\mathcal{C}} n_{\bar{c}} E(\bar{c})}{\sum_{\bar{c}\in\mathcal{C}} n_c} + \sum_{c\in\mathcal{C}} n_c \frac{f(y,n)}{\sum_{\bar{c}\in\mathcal{C}} n_{\bar{c}}} = f(y,n).$$

Since revenue equals cost for all equilibrium and potential jurisdictions, it follows that for all potential crowding profiles $\bar{n} \in N$ and all public goods levels $\bar{y} \in \Re^L_+$,

$$\sum_{\{c\in\mathcal{C}|\bar{n}_c>0\}} \bar{n}_c \hat{\rho}_c(\bar{y},\bar{n}) - f(\bar{y},\bar{n}) \leq 0.$$

and for all $s^k \in S$,

$$\sum_{i\in s^k} \hat{\rho}_{A(i)}(y^k, CP(A,s)) - f(y^k, CP(A,s)) = 0.$$

It only remains to show that it is optimal for agents to choose to participate in the core state under these prices. Suppose instead that for some individual $i \in s^k \in S$ where $\tau(i) = t$, there exists a crowding profiles $\bar{n} \in N$, a crowding assignment $\hat{A} \in \mathcal{A}$ and levels of public good production $\bar{y} \in \Re^L_+$ such that

$$\omega_t - E(A(i)) - \hat{\rho}_{A(i)}(y^k, CP(A,s^k)) + h_t(y^k, CP(A,s^k)) <$$

$$\omega_t - E(\hat{A}(i)) - \hat{\rho}_{\hat{A}(i)}(\bar{y},\bar{n}) + h_t(\bar{y},\bar{n}).$$

First we claim that agent $i \in \mathcal{I}$ who is of type $t \in \mathcal{T}$ is indifferent over all possible choice of crowding type represented in the crowing profile $\bar{n}$, given that he joins such a coalition. Formally, we must show that for all $c, \hat{c} \in \mathcal{C}$ such that $\bar{n}_c > 0$ and $n_{\hat{c}} > 0$, it holds that

$$\omega_t - E(c) - \hat{\rho}_c(\bar{y}, \bar{n}) + h_t(\bar{y}, \bar{n}) = \omega_t - E(\hat{c}) - \hat{\rho}_{\hat{c}}(\bar{y}, \bar{n}) + h_t(\bar{y}, \bar{n}).$$

Subtracting the common terms and substituting in for prices, this is equivalent to showing:

$$-E(c) - \left[\frac{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}} E(\bar{c})}{\sum_{\bar{c} \in \mathcal{C}} n_c} - E(c) + \frac{f(y,n)}{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}}} \right] =$$

$$-E(\hat{c}) - \left[\frac{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}} E(\bar{c})}{\sum_{\bar{c} \in \mathcal{C}} n_c} - E(\hat{c}) + \frac{f(y,n)}{\sum_{\bar{c} \in \mathcal{C}} n_{\bar{c}}} \right],$$

which is obviously true.

Consider an coalition $\bar{s}$ consisting of $\sum_{\bar{c} \in \mathcal{C}} \bar{n}_{\bar{c}}$ agents of type t. By construction of the price system, only coalitions with B or fewer agents are available through the markets to agents, we know that $\sum_{\bar{c} \in \mathcal{C}} \bar{n}_{\bar{c}} \leq B$ and by SGE, there are at least B agents of type t. It follows that $\bar{s}$ is a feasible jurisdiction. Let $\bar{A} \in \mathcal{A}$ be any assignment such that $CP(\bar{A}, \bar{s}) = \bar{n}$. Since we have already shown that agents of type t are indifferent over all possible crowding type assignments under the proposed prices when they join the jurisdiction $\bar{s}$, and by Lemma 1, all agents are of type t are equally treated in the core, we know that for all $j \in \bar{s}$,

$$\omega_t - E(A(j)) - \hat{\rho}_{\bar{A}(j)}(\bar{y}, CP(\bar{A}, \bar{s})) + h_t(\bar{y}, CP(\bar{A}, \bar{s})) =$$

$$\omega_t - E(\hat{A}(i)) - \hat{\rho}_{\hat{A}(i)}(\bar{y}, \bar{n}) + h_t(\bar{y}, \bar{n}) >$$

$$\omega_t - E(A(i)) - \hat{\rho}_{A(i)}(y^k, CP(A, s^k)) + h_t(y^k, CP(A, s^k)).$$

Since we show above that the costs of the public goods are covered under these prices, the allocation is feasible for $\bar{s}$ and this coalition improves upon the core state (X, Y, A, S), a contradiction.

We conclude that under these prices, cost are covered, profits are non-positive, and agents can do no better than by choosing to participate in the core state. Therefore, these prices decentralized the core.

■

It is an immediate corollary of Theorem 3 and 4 that the core and equilibrium are equivalent.

5. Conclusion

Making a formal distinction between the tastes of agents and their external crowding effects allows us to address some long-standing questions in local public economics. Most importantly, it makes it possible to confirm Tiebout's assertion that efficient allocations can be decentralized through anonymous prices even when crowding is differentiated. It also creates some new questions. In the crowding types model, it is possible to form coalitions that take advantage of the full array of crowding effects while segregating agents with according to tastes. Both our intuition and everyday experience suggest that such taste-homogeneous coalitions should be able to out-perform any taste-heterogeneous coalition with the same crowding profile. It is surprising, therefore, that not only is this not true in general, but that it may indeed be generically false. Only when crowding types appear in exactly the right proportion can we be assured that taste-homogeneous coalitions will be optimal.

The main difference between this paper and our previous work is that we make crowding type an endogenous variable. Modeling crowding type as an exogenous characteristic of agents is a significant restriction which limits its interpretation to things like genetic endowments and external effects resulting from irreversible choices. Allowing crowding type to be endogenous opens the model up to a much richer set of interpretations. The most interesting in our opinion is to think of crowding type as representing the skills that agents acquire as a result of their educational investment choices. We show that in such an environment market forces cause agents to choose the various types of skills in exactly the proportions that imply that the optimal coalitions will be essentially taste-homogeneous. Thus, to the degree that agents choose their crowding characteristics in response to market signals, agents will find it advantageous to segregate according to taste even when crowding is differentiated.

We also show that the core and equilibrium are equivalent when crowding type is endogenous and that decentralization is possible with anonymous prices. This conflicts with results for the standard differentiated crowding model in which decentralizing prices must be nonanonymous, but agrees with results for the crowding types model. The endogeneity of crowding types, however, lets us say more about the structure of the price system than we were able to say in the exogenous case. Specifically, we show that, the net an agent's own educational costs, each agent pays exactly the average cost of public goods production plus the average of all of his fellow members' education costs when join a particular jurisdiction. This is highly intuitive result. It says that agents who spend a lot on education must be compensated by agents who economize so that in the end, all agents pay the same net education costs. In addition, all agents pay an equal share of the cost of producing public goods. Since the core is essentially taste-homogeneous, this is equivalent to what they would pay at a Lindahl equilibrium.

This paper makes a number of simplifying assumptions in order to make the proofs transparent and highlight the new results this model provides. Obtaining asymptotic versions of these results with the model generalized to many private goods and ordinal preferences appears to be primarily a technical exercise. This is especially true in view of the fact that convergence of approximate cores to equilibrium outcomes in economies with local public goods has already been shown (cf. Wooders (1981, 1989,1996)).

We have focused on the core. It has been well-known since Pauly (1970) that cores of economies with local public goods may be empty. In fact, for economies in general, except under certain stylized conditions, cores are typically empty. The rationale for our interest in the core is that, in large economies with small effective groups, approximate cores are typically nonempty, are approximately symmetric,[10] and approximate cores converge to equilibrium payoffs.[11] Thus we assume, for our results, that there is some state of the economy in the core. If there is a cost of coalition formation, or some "market friction", then given a jurisdiction structure, improving coalitions may have to pay a cost of forming and thus not be able to achieve the same payoffs as identical coalitions in the given jurisdiction structure. Under this definition of the core Wooders (1988) shows that with ordinal preferences, large economies with small effective groups where improving coalitions must pay a set-up cost have nonempty cores.[12]

There are several substantive ways that this model might be generalized. We have assumed that all agents face the same educational costs. Agents we commonly observe, however, may face vastly different costs of acquiring skills. Different people have different aptitudes. This could be addressed by including a second characteristic in the definition of agents (in addition to an agents tastes) which we might call "ability". Educational costs would then depend both and an agents ability and the educational choice. We have also assumed that agents care only about the crowding profile of the jurisdiction they join, and not about the particular skill they end up choosing. Clearly, people care a great deal about the type of work they do. This observation could be incorporated into the model by generalizing the preference function to include the agent's choice. It is not immediately clear how these two modifications would effect the results in this paper.

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[10] That is, most agents of each type are treated approximately equally in terms of the payoffs that they receive.

[11] See Wooders (1980,1983), Wooders and Zame (1984) and other papers for nonemptiness of approximate cores, Wooders (1980,1994a) for the equal treatment property of the core and asymptotic equal treatment, and Wooders (1980), Wooders and Zame (1987) for convergence of approximate cores. A lengthy survey appears in Wooders (1994a) while Kannai (1992) provides a short survey.

[12] Another definition of approximate cores ignores an "exceptional set" of agents, consisting of at most a small fraction (ϵ) of the economy. Shubik and Wooders (1983a,b) show that using this definition of an approximate core, only the very mild assumption! of boundedness of the set of equal treatment payoffs (analogous to per capita boundedness of payoff in a setting with quasi-linear preferences) is required to obtain nonemptiness of approximate cores of large economies. The difficulty with this notion is that it is not easy to justify why a percentage of players can be ignored.

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