Coalitions and Clubs:†
Existence, Decentralization, and Optimality

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1. Introduction

A central problem in public economics is how to achieve optimal outcomes through price-based mechanisms in economies with public goods. One of the key papers is Samuelson (1954) where Lindahl's approach is given a formal statement. Unfortunately, the implied system of personalized prices requires agents to truthfully reveal their preferences for public goods. As Samuelson notes, an agent therefore may have an incentive to misrepresent his true preferences. Thus, it is doubtful that market mechanisms based on the Lindahl prices defined by Samuelson would generally be able to efficiently provide public goods.

In response, Tiebout (1956) argued that many types of public goods are subject to crowding and congestion, resulting in the possibility of their being provided by local jurisdictions rather than national governments. Jurisdictions offering consumers various bundles of public goods can condition residence in the jurisdiction (and therefore consumption of the public goods) on the payment of taxes. Thus, agents in effect are forced to reveal their willingness to pay for public goods through their locational choice. Tiebout asserted that, in "large" economies when localities compete for residents and agents, in turn, "vote with their feet" to express public goods demand, such goods will be efficiently provided.

Tiebout (1956) stimulated a large theoretical investigation. Subsequent researchers have shown that, although efficient Tiebout sorting may not occur in completely general circumstances, adding economic restrictions which are natural in the study of clubs or local public goods provide support for Tiebout's hypothesis. For example, Wooders (1978) showed that when congestion is anonymous (only the number of agents sharing the public good matters) and all gains to coalition-forming are realized in small coalitions, the core can be decentralized with anonymous admission prices.[1] Bewley (1981) is sometimes cited as an early attempt to formalize the Tiebout hypothesis and his results were not promising. He showed that, in some cases, anonymous decentralization

[1] While Wooders (1978) was an early result, many other works, for example Wooders (1980), also provide support for the Tiebout hypothesis. These works will be surveyed throughout this chapter.

of efficient outcomes is not possible and, in other cases, anonymous prices may only decentralize inefficient outcomes. However, in Bewley (1981) the number of jurisdictions is fixed and public goods are note subject to congestion. And so, in important ways, Bewley (1981) does not represent a Tiebout model in the way the literature has come to accept. This notwithstanding, Bewley makes a key point that any meaningful interpretation of Tiebout's hypothesis must embed the idea that that decentralizing prices are anonymous in the sense that do not depend on agents' unobservable characteristics. We will have more to say about this below.

The *local public goods* approach to the provision of congestible public goods centers on an agent's locational choice among competing jurisdictions which offer distinct public good bundles. Broadly taken, local public goods models address the general equilibrium question of how to best sort the entire population of a large economy into non-overlapping and exhaustive coalitions. Alternatively, the *club* approach to the provision of congestible public goods are usually partial equilibrium models of private membership organizations such as a country clubs or private schools. Club models generally address the profit-maximizing decisions of a club with price-taking members. Early models of tolls and congested roads by Pigou (1920) and Knight (1924) are considered club models. However, Buchanan (1965) was the first to develop a formal club model and is often credited with beginning the modern club literature. It should be noted that the language which we use to distinguish clubs from local public goods is not universal. Berglas, Helpman, and Pines (1982), Sandler and Tschirhart (1980), and Cornes and Sandler (1996) offer distinct taxonomies. For example, Cornes and Sandler refer to local public goods economies as club economies whose population is partitioned.

The club literature developed in myriad directions since Buchanan published his seminal paper. Pauly (1967, 1970) explored the issue of optimal club size and the stability of its membership. Tollison (1972), Ng and Tollison (1974), Berglas (1976), and DeSerpa (1977) present clubs in which crowding is nonanonymous.[2] Wooders (1978)

[2] The early authors did not use the terminology "nonanonymous crowding," that is the more modern

considered anonymous prices, prices which do not depend on unobservable characteristics of agents. McGuire (1974) and Pauly (1970) address whether club membership will be homogeneous when agents differ in tastes or endowments. Questions of core and equilibrium existence in club economies arose in works such as Pauly (1967,1970) and Wooders (1978). Issues of costs for excluding unwanted members of an exclusive club are presented by Davis and Whinston (1967), Millward (1970), Nichols *et al.* (1971), Oakland (1972), and Kamien *et al.* (1973). And early explorations of uncertainty in clubs models include DeVany and Saving (1977) and Hillman and Swan (1979). Cornes and Sandler (1996) survey each of these expansive research areas and provide an introduction to more recent directions of investigation such as multiproduct, variable usage, and intergenerational clubs.

Our purpose in this chapter is to draw attention instead to a recent approach to formalizing Tiebout's central insight in the local public goods tradition, the crowding-types model introduced in Conley and Wooders (1997a). The remainder of this paper is organized as follows. In Section 2 we introduce the agents who populate the local public goods economy presented. In Section 3 we discuss various forms of crowding which have been studied. In Section 4, optimality and decentralization results are given. In Section 5, existence results are presented. Section 6 discusses non-cooperative solution concepts and Section 7 discusses other results and future research.

2. Agents and Crowding

We consider an economy with I agents, each of whom resides in exactly one jurisdiction, consumes one private good, $x \in \Re$, and M local public goods $y \in \Re^M_+$.[3]

language used to describe congestion caused by agents' characteristics rather than the club size.

[3] Until we consider models with linear price systems, the Euclidean structure of the public goods space is not used. Thus, it is trivial to generalize this to consider an abstract set of public projects. It is not as trivial to generalize this to multiple private goods, although the results carry through to continuum economies in this case, and hold approximately for finite economies.

The set of agents is given by $\mathcal{I} \equiv 1, \ldots, i, \ldots, I$ where $i \in \mathcal{I}$ is an individual. In this economy, agents will typically find it optimal to form many small jurisdictions for the purpose of consuming local public goods. Therefore, the coalitional structures we study are partitions. An arbitrary jurisdiction of agents is denoted $s \subset \mathcal{I}$ and $\mathcal{S}$ denotes the set of all possible jurisdictions. A list of jurisdictions $s^1, \ldots, s^P \equiv S$ is a *partition* if $\cup_p s^p = \mathcal{I}$ and $s^p \cap s^{\bar{p}} = \emptyset$ for all $s^p, s^{\bar{p}}$ such that $p \neq \bar{p}$.

An agent possesses one of T different types of tastes or preferences, indexed by $t \in 1, \ldots, T \equiv \mathcal{T}$. The mapping $\tau : \mathcal{I} \to \mathcal{T}$ assigns a taste type and a corresponding endowment, ω_t, to each agent in the economy. Thus, if agent i possesses taste type t, then $\tau(i) = t$. It will sometimes be useful to describe the population of a jurisdiction by the number of agents of each taste type in that jurisdiction. The total population of agents in the economy is given by $N = (N_1, \ldots, N_t, \ldots, N_T)$ where N_t is interpreted as the number of agents of type t in the economy. Similarly, a jurisdiction s has population $N^s = (N_1^s, \ldots, N_t^s, \ldots, N_T^s)$ where N_t^s is the number of agents of type t in jurisdiction s.

Agents are also distinguished by their crowding characteristics. A crowding type captures all elements of an agent that generate external effects on other agents who share the same coalition membership. These effects could be positive or negative, and we will assume that they are publicly observable. For example, one's gender, race, height, or intelligence may confer costs or benefits to one's coalition-mates. We will explore this in more detail below.

Formally, each agent is classified as having one of C publicly observable crowding types, indexed by $c \in 1, \ldots, C$. The mapping $\kappa : \mathcal{I} \to \mathcal{C}$ associates a crowding type with each agent in the economy.[4] That is, $\kappa(i) = c$ indicates that agent i possesses crowding characteristic c. Let K be the set of all possible such mappings. Denote a profile of crowding characteristics by $n = (n_1, \ldots, n_c, \ldots, n_C) \in Z_+^C$, where Z is the

[4] It should be noted that in the specifications that follow, this mapping will sometimes be exogenously given and will sometimes be endogenous.

set of integers and n_c is interpreted to be the number of agents of type c. For any assignment of agents to crowding types $\kappa \in K$, the *crowding profile of a jurisdiction* $s \in \mathcal{S}$ is the mapping $\mathcal{K} : K \times \mathcal{S} \to Z_+^C$ given by

$$\mathcal{K}(\kappa, s) \equiv (\parallel s_1 \parallel , \ldots , \parallel s_C \parallel),$$

where $i \in s_c$ if and only if $i \in s$ and $\kappa(i) = c$, and where $\parallel \cdot \parallel$ denotes the cardinality.

The commonly available public goods production technology is given by the cost function $f : \Re_+^M \times Z_+^C \to \Re$ where

$$f(y, n)$$

is the cost in terms of private goods of producing the public goods levels y for a jurisdiction with crowding profile given by n. Note that crowding effects are also permitted. For example, it may be cheaper to construct pubic buildings if agents are of a type which is less likely to commit vandalism, and it may be easier to establish a good departmental research reputation if the department is populated by smart and hard working types.

One focus of these coalitional economy models is to explore the form of crowding or congestion effects. Crowding can take many forms. Here we consider anonymous crowding, differentiated crowding, exogenous crowding types, endogenous crowding types where agents have equal abilities, and endogenous crowding types where agents have differentiated genetic endowments. In the following subsections, we describe these various forms of congestion.

2.1 Anonymous Crowding

When crowding is anonymous, each agent cares only about the number of individuals in his jurisdiction. For example, when standing in line at the movie theater, only the length of the queue matters, not the theater-goers' heights, professions, or even their tastes in movies. Formally, anonymous crowding implies that there is only one exogenously given crowding type, $\kappa(i) = c$ for all $i \in \mathcal{I}$, and the crowding profile of a jurisdiction, $\mathcal{K}(\kappa, s) = (\parallel s_c \parallel)$, is simply the number of agents in the jurisdiction. An agent i has a utility function $u^A_{\tau(i)} : \Re \times \Re^M_+ \times Z_+ \rightarrow \Re$ given by $u^A_t(x_i, y, \parallel s \parallel)$, where x_i is agent i's private good level, s is the jurisdiction in which i resides, and y is the public goods levels in jurisdiction s. Note that this specification of preferences ensures that agents are only affected by the number of people sharing the public good and not by their taste types. Anonymous crowding may also have an effect on the cost of public goods production. In this case, the cost in terms of private good of producing the public goods bundle y in jurisdiction s may be written $f^A(y, \parallel s \parallel)$. Models of local public goods in which crowding is anonymous include Wooders (1978, 1980), Berglas and Pines (1980), Boadway (1980), Scotchmer and Wooders (1987), and Barham and Wooders (1998). Barham and Wooders (1998) provide a survey of the many other contributions in which crowding is anonymous.

2.2 Differentiated Crowding

Often agents care about the types of agents who share residence in their jurisdiction, not just the number of people residing there. For example, when dining out, the numbers of smokers and non-smokers affect the enjoyment of dinner. In this case, an agent's unobservable preferences, whether he enjoys smoking in restaurants, may be perfectly correlated with an observable crowding characteristic, whether he actually smokes in the restaurant. Thus, if crowding is differentiated, an agent's crowding type is said to be exogenously given and perfectly correlated with tastes. As a result, the crowding type index, $c \in \mathcal{C}$, and the taste type index, $t \in \mathcal{T}$, may be used interchangeably and a jurisdiction's crowding profile is simply its population, $\mathcal{K}(\kappa, s) = (\parallel s_1 \parallel, \ldots, \parallel s_c \parallel, \ldots, \parallel s_C \parallel) = (N^s_1, \ldots, N^s_t, \ldots, N^s_T)$. An agent i has utility function $u^D_{\tau(i)} : \Re \times \Re^M_+ \times Z^T_+ \rightarrow \Re$ given by $u^D_t(x_i, y, (N^s_1, \ldots, N^s_t, \ldots, N^s_T))$, where x_i is the level of private goods, s is the jurisdiction in which i resides, and y is the public goods levels in jurisdiction s. If differentiated crowding is manifest in the public goods production, then the cost in terms of private good of producing the public goods bundle y in jurisdiction s can be written $f^D(x, y, (N_1^s, \ldots, N_t^s, \ldots, N_T^s))$. Early studies of differentiated crowding naturally arose in the context of labor complementarities and peer group effects. Notable contributions along those lines include Berglas (1976) in which differentiated crowding was introduced, de Bartolome (1990), McGuire (1991), Schwab and Oates (1991), Benabou (1993), and Brueckner (1994). General equilibrium local public good models in which crowding is differentiated are formalized in Wooders (1981,1989), Scotchmer and Wooders(1986), and Scotchmer(1994).

2.3 Exogenous Crowding Types

In an important way, differentiated crowding is an unappealing generalization of the anonymous crowding model. In general, it would be somewhat surprising if agents' unobservable tastes were perfectly correlated with their observable crowding types. For example, at a dance club, an agent's gender is an observable crowding characteristic. For any individual, one gender crowds positively while the other is simply in the way. Both genders, however, have some members who prefer rock music to jazz and others who prefer jazz to rock. Tastes in this case are unobservable and unrelated to how agents crowd each other. To capture this formally, utility and cost functions are allowed to depend on a jurisdiction's crowding profile but not on agents' tastes. We assume, as above, that an agent's crowding type is exogenously assigned; that is, we assume that the mapping κ is fixed and known by all agents. An agent i has a utility function $u_{\tau(i)}^{EX} : \Re \times \Re_+^M \times Z_+^C \to \Re$, given by $u_t^{EX}(x_i, y, n)$, where x_i is the private goods level, y is the public goods levels, and n is the crowding profile in the jurisdiction in which i resides. Similarly, when only crowding characteristics matter, the cost in terms of private good of producing the public goods bundles y in jurisdiction s can be written $f^{EX}(y, n)$. Conley and Wooders (1997a) introduce this crowding types model with other results included in Conley and Wooders (1997b), Conley and Wooders (1998), and Conley and Smith (2003).

2.4 Endogenous Crowding Types, Equal Abilities

The exogenous crowding types model is most appropriately applied to situations in which agents have no control over their crowding types. For example, agents have no control over race, height, national origin, basic mental abilities, and very little control over others such as gender, looks, charisma, and so on. An exogenous crowding types approach may also be appropriate when an agent's crowding type results from a past irreversible decision such as learning a second language. However, many observable characteristics that affect the welfare of others are the result of choices made in response to clear market signals. For example, when a graduate student chooses a major field, the knowledge of demand for the various types of economists certainly plays a role in the decision. Crowding characteristics such as profession, marital status and number of children, and level participation in the community are chosen at least partly in response to costs and benefits they expect to receive for their choices.

To capture this, suppose that an agent may choose, subject to the constraints of educational costs, his own observable crowding characteristics. The cost to any agent of choosing to become a given crowding type is given by the mapping $\epsilon : \mathcal{C} \to \Re$, called the *educational cost function.* For example, $\epsilon(c)$ gives the educational cost that an agent must incur to obtain crowding type c. Note that agents are "equally able" in that they all face the same educational cost of obtaining a particular skill. An agent i has a utility function $u^{EN}_{\tau(i)} : \Re \times \Re^M_+ \times K \times Z^C_+ \to \Re$, given by $u^{EN}_{\tau(i)}(x_i - \epsilon(c), y, c, n)$, where x_i is the gross private goods level, c is the chosen crowding characteristic, and y and n are the public goods levels and crowding profile in the jurisdiction in which i resides. Note that each agent pays the full cost of his education and that $x_i - \epsilon(c)$ is his private good consumption net of education costs. Note also that "c" is an argument in the utility function. This is to capture the idea that agents my have preferences over which crowding type they acquire. For example, I may wish to be a rock star or a poet, but choose the dreary life of an economist because of the expected wages, and because it is, after all, preferred to being an accountant. Of course, the crowding profile of a jurisdiction may also affect production. In this case, the cost in terms of private good of producing the public goods bundles y in a jurisdiction with crowding profile n can be written $f^{EN}(y, n)$. This model was introduced in Conley and Wooders (1996).

2.5 Endogenous Crowding and Differential Genetic Types

Often, an agent's genetic abilities affect the cost of acquiring a particular skill crowding type. For example, a 7 foot man is likely to have a lower cost of becoming a professional basketball player than a 5 foot man does. An intelligent person may find it easy get a Ph.D. in physics but may have no facility for typing. Note also that we would not generally expect that an agent's preferences to related to his genetic endowments. For example, many professional athletes would rather be movie stars and many movie stars would rather be politicians.

Agents in this case are described by their preferences and genetic endowments. We assume these are uncorrelated and not publicly observable. Each agent may choose, subject to the constraints of educational costs, his own observable crowding characteristic. Only an agent's crowding type directly affects the welfare of others. An agent's educational cost of obtaining a particular crowding characteristic depends on his genetic endowment. Again, independent of this cost, individuals have preferences over which skill is acquired.

Formally, each agent $i \in \mathcal{I}$ is described by his preferences, $\tau(i)$, and one of G different types of genetic endowments, indexed by $g \in 1, \ldots, G \equiv \mathcal{G}$. The mapping $\gamma : \mathcal{I} \to \mathcal{G}$ exogenously ascribes a genetic endowment to each agent in the economy. Each agent chooses to acquire one of the c publicly observable crowding characteristics. Agents with different genetic endowments may have different abilities and, therefore, face different costs of acquiring a given crowding type. In this case, the *educational cost function* is given by $\epsilon_{\gamma(i)} : \mathcal{C} \to \Re$, where $\epsilon_{\gamma(i)}(c)$ gives the cost to agent i of obtaining crowding characteristic c. Note that this cost may be negative as some types of education may generate income.

An agent i has a utility function $u_{\tau(i)} : \Re \times \Re^M_+ \times K \times Z^C_+ \to \Re$, given by

$$u_{\tau(i)}(x_i - \epsilon_{\gamma(i)}(c), y, c, n),$$

where x_i is the gross private good level, c is the chosen crowding characteristic, and y and n are the public goods levels and crowding profile of the jurisdiction in which i resides. As above, the agent pays the full cost of his education and so $x_i - \epsilon_{\gamma(i)}(c)$ is the private goods consumption level net of education costs. Crowding is treated in the standard way.

This most recent approach to modeling crowding characteristics was introduced by Conley and Wooders (2001). It is easy to see that each of the other approaches described above can be obtained as special cases. For example, if each agent has the same genetic type and the same costlessly acquired crowding characteristic, the model becomes the anonymous crowding model. Suppose, instead, that agents all have the same genetic type, there are the same number of crowding types as taste types ($T = C$), that the cost of acquiring any crowding characteristic is zero, and that an agent with taste type t always prefers to be crowding type $c = t$ rather than any other $\bar{c} \neq t$. Then, in equilibrium, all agents sharing a taste type will choose the same crowding type; taste and crowding characteristics are perfectly correlated. The special case described is the differentiated crowding model. To obtain the exogenous crowding types model, suppose that the number of genetic types, G, is equal to the number of crowding types C. Furthermore, for genetic type g the cost of obtaining crowding type $c = g$ is zero and the cost of obtaining any other crowding type $\bar{c} \neq g$ is greater than the private goods endowment. Then, in equilibrium, the genetic and crowding characteristics are perfectly correlated – in effect, the crowding characteristic is exogenously identified with the genetic characteristic. Finally, to obtain the endogenous crowding type model with equal abilities, suppose all agents possess the same genetic type and they are all indifferent over their choices of crowding characteristic. The fact that these different crowding forms can be stated as special cases of the endogenous crowding with differential genetic abilities allows us to state definitions and results in terms of this most general case.

2.6 Extensions

Thus far, only crowding effects that relate to an agent's coalition membership have been considered. Clearly, this approach is too simple for some economic situations. The classic cases of toll roads and athletic clubs are other examples of situations in which an agent's utility can depend not only on the size of a facility, y, and the crowding profile of those with whom he shares, n, but also on the number of visits he make to the club, v_i and as well as the total number of visited made by each crowding type $(V_1, \ldots, V_c, \ldots, V_C)$. For example, Arnott and Kraus (1998) conjoin the urban transportation and congestible facilities models with the club literature in a model with crowding and anonymous prices. We could also consider how, in a coalitional economy in which agents choose how to distribute their endowments across various coalitions, production technology may be affected. For example, the bright colleague who never shows up for seminars creates few positive externalities. On the other hand, the annoying colleague who always shows up to faculty meetings creates many negative external effects.

Alternatively, we could consider a model of local consumption externalities. It may be that external effects flow not from an exogenous or endogenous characteristic, but through the actions of agents. For example, I don't care whether you enjoy smoking (a taste), or even whether you are a smoker (a crowding characteristic). What I do care about is how many cigarettes you smoke in my presence. Similarly, I care about whether you renovate your house because it affects my property values. To our knowledge, these effects have not been thoroughly explored in coalitional economies.

3. A Local Public Goods Economy

In this section we define a local public goods economy and the core and provide theorems for the equal treatment property of the equal implicit contributions of core allocations.

A *feasible state of the economy* is a list,

$$(X, Y, \kappa, S) \equiv ((x_1, \dots, x_I), (y^1, \dots, y^P), \kappa, (s^1, \dots, s^p, \dots, s^P)),$$

where X is an allocation of private good for each agent, Y gives level of each public good for each jurisdiction, κ is an assignment of agents to crowding types, and S is a partition of the population, such that

$$x_i - \epsilon_{\gamma(i)}(\kappa(i)) \geq 0, \quad \text{for all } i \in \mathcal{I},$$

and

$$\sum_{i \in \mathcal{I}} (\omega_{\tau(i)} - x_i) - \sum_p f(y^p, \mathcal{K}(\kappa, s^p)) \geq 0.$$

We denote the set of feasible states as F. A pair $(\bar{x}, \bar{y})$ is a *feasible allocations for a jurisdiction $\bar{s}$ under crowding assignment $\bar{\kappa}$* if

$$\bar{x}_i - \epsilon_{\gamma(i)}(\bar{\kappa}(i)) \geq 0, \quad \text{for all } i \in \mathcal{I},$$

and

$$\sum_{i \in \bar{s}} (\omega_{\tau(i)} - \bar{x}_i) - \sum_p f(\bar{y}, \mathcal{K}(\bar{\kappa}, \bar{s})) \geq 0.$$

We denote the set of feasible allocations for a jurisdiction, $\bar{s}$, under crowding assignment, $\bar{\kappa}$ as $A(\bar{s}, \bar{\kappa})$.

A jurisdiction $\bar{s} \in \mathcal{S}$ under crowding assignment $\bar{\kappa}$ producing $(\bar{x}, \bar{y}) \in A(\bar{s}, \bar{\kappa})$ *improves upon* a feasible state $(X, Y, \kappa, S) \in F$ if, for all $i \in \bar{s}$,

$$u_{\tau(i)}[\bar{x}_i - \epsilon_{\gamma(i)}(\bar{\kappa}(i)), \bar{y}, \bar{\kappa}(i), \mathcal{K}(\bar{\kappa}, \bar{s})] \geq u_{\tau(i)}[x_i^p - \epsilon_{\gamma(i)}(\kappa(i)), y^p, \kappa(i), \mathcal{K}(\kappa, s^p)],$$

where $i \in s^p \in S$ in the original feasible state, and for some $j \in \bar{s}$,

$$u_{\tau(j)}[\bar{x}_j - \epsilon_{\gamma(j)}(\bar{\kappa}(j)), \bar{y}, \bar{\kappa}(j), \mathcal{K}(\bar{\kappa}, \bar{s})] > u_{\tau(j)}[x_j^{\hat{p}} - \epsilon_{\gamma(j)}(\kappa(j)), y^{\hat{p}}, \kappa(j), \mathcal{K}(\kappa, s^{\hat{p}})],$$

where $j \in s^{\hat{p}} \in S$ in the original feasible state. A feasible state $(X, Y, \kappa, S) \in F$ is a *core state* if it cannot be improved upon by any allocation. The *core* of the economy is the set of all core states.

What drives an economy to display the localness of public goods provision suggested by Tiebout is that all or almost all of the gains from forming coalition can be realized by groups that are small compared to the size of the population. Tiebout (1956) suggested that seven assumptions would guarantee that efficient allocations in a local public goods economy could be decentralized. His assumption six, "[f]or every pattern of community services...there is an optimal community size," informally defines a local public goods economy. There are several alternative formulations of the idea of an optimal group size including exhaustion of blocking opportunities, per capita boundedness, distinguished numbers, and minimum efficient scale. See Conley and Wooders (1997b) for a more detailed discussion of these alternatives. In this chapter, we use the idea of *strict small group effectiveness* to formalize Tiebout's assumption six. Strict Small Group Effectiveness is a variant of exhaustion of gains to scale or "minimum efficient scale" from Wooders (1983). Formally, an economy satisfies *strict small group effectiveness,* (SSGE), if there exists a positive integer $B \in Z_+$ such that

1. for all core states (X, Y, κ, S) and for all $s^p \in S$ it holds that $\parallel s^p \parallel \; \leq B$;
2. if a feasible state (X, Y, κ, S) can be improved upon, then there exists a coalition $\bar{s} \in \mathcal{S}$ such that $\parallel \bar{s} \parallel \; \leq B$ which can also improve upon (X, Y, κ, S);
3. for all $t \in \mathcal{T}$ and $g \in \mathcal{G}$, either $\parallel i \in \mathcal{I} \parallel \tau(i) = t \text{ and } \gamma(i) = g \parallel \; > B$ or $i \in \mathcal{I} \parallel \tau(i) = t \text{ and } \gamma(i) = g = \emptyset$.

The first condition states that core state jurisdictions are "small," or bounded in size. The second condition states that all possibilities to improve upon a state are realized in small coalitions. And the last condition guarantees that if any type of agent is actually represented in the economy, then there must be enough agents of that type to populate that largest potentially optimal jurisdiction. Note that in the one private good case, a jurisdiction improves upon a feasible state without trading private goods outside the jurisdiction because no gains from such trade are possible. One interpretation of this model is that there are many unspecified private goods whose prices are taken as given and the one specified private good is the economy's "money." If the economy included multiple private goods, gains from trading among jurisdictions would be possible.

An immediate consequence of *strict small group effectiveness* is the equal treatment property of the core. This follows from (Wooders 1983, Theorem 3) and is proven specifically for this economy in Conley and Wooders (2001). Theorem 1 states that any two agents with the same taste, crowding, and genetic type are treated equally in the core.

Theorem 1. *Let (X, Y, κ, S) be a core state of an economy satisfying $SSGE$. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\tau(i) = \tau(\hat{i})$ and $\gamma(i) = \gamma(\hat{i})$, if $i \in s^p$ and $\hat{i} \in s^{\hat{p}}$, then*

$$u_t(x_i - \epsilon_g(\kappa(i)), y^p, \kappa(i), \mathcal{K}(\kappa, s^p)) = u_t(x_{\hat{i}} - \epsilon_g(\kappa(\hat{i})), y^{\hat{p}}, \kappa(\hat{i}), \mathcal{K}(\kappa, s^{\hat{p}})).$$

A second consequence of *strict small group effectiveness* is that agents residing in two jurisdictions with the same crowding profile and public goods levels who each choose to become the same crowding type must make the same implicit contributions to the jurisdiction for its public goods productions. That result, given in the following theorem, is important because it implies that implicit decentralizing prices are anonymous in that they do not depend on agents' tastes. This theorem is proven in Conley and Wooders (2001).

Theorem 2. *Let (X, Y, κ, S) be a core state of an economy satisfying SSGE, and let $s^p, s^{\hat{p}} \in S$ be a pair of jurisdictions in the core partition such that $y^p = y^{\hat{p}}$ and $\mathcal{K}(\kappa, s^p) = \mathcal{K}(\kappa, s^{\hat{p}})$. Then for any crowding type $c \in \mathcal{C}$ and any pair of agents $i \in s^p$ and $\hat{i} \in s^{\hat{p}}$ such that $\kappa(i) = \kappa(\hat{i}) = c$, it holds that*

$$\omega_{\tau(i)} - x_i - \epsilon_{\gamma(i)}(c) = \omega_{\tau(\hat{i})} - x_{\hat{i}} - \epsilon_{\gamma(\hat{i})}(c).$$

4. Optimality and Decentralization

In this section we define a price-taking equilibrium concept and provide core equivalence results. Let $\mathcal{K}_c$ denote the set of crowding profiles which include at least one agent of crowding type c:

$$\mathcal{K}_c \equiv \left\{n \in Z_+^C \mid \mathcal{K}(\kappa, s) = n \text{ for some } s \in \mathcal{S} \text{ and } \kappa(i) = c \text{ for some } i \in s.\right\}$$

An anonymous admission price system for an agent possessing type c is given by the mapping $\rho_c : \Re_+^M \times \mathcal{K}_c \to \Re$ where $\rho_c(y, n)$ is the price that an agent with crowding type c would have to pay to join a jurisdiction producing public goods levels y and having a crowding profile $n \in \mathcal{K}_c$. We call this system anonymous because prices depend only on observable characteristics (crowding types) and not on any private information (tastes or genetic endowments). A *Tiebout admission price system*, ρ, is a collection of such price systems, one for each crowding type.

A *Tiebout equilibrium* is a feasible state $(X, Y, \kappa, S) \in F$ and a Tiebout admission price system, ρ, such that:

1. for all $s^p \in S$, all individuals $i \in s^p$, any alternative crowding profile $\bar{n} \in Z_+^C$, all alternative crowding choices c satisfying $\bar{n} \in \mathcal{K}_c$, and for all alternative public goods levels, $\bar{y} \in \Re_+^M$,

$$u_{\tau(i)}[\omega_{\tau(i)} - \rho_{\kappa(i)}(y^p, \mathcal{K}(\kappa, s^p)), y^p, \kappa(i), \mathcal{K}(\kappa, s^p)] \geq u_{\tau(i)}[\omega_{\tau(i)} - \rho_c(\bar{y}, \bar{n}), \bar{y}, c, \bar{n}];$$

2. for all potential jurisdictional crowding profiles $\bar{n} = (\bar{n}_1, \ldots, \bar{n}_c \ldots, \bar{n}_C) \in Z_+^C$ and public goods levels $\bar{y} \in \Re_+^M$,

$$\sum_{\{c \in \mathcal{C} \ \| \ \bar{n}_c > 0\}} \bar{n}_c \rho_c(\bar{y}, \bar{n}) - f(\bar{y}, \bar{n}) \leq 0;$$

3. and, for all $s^p \in S$,

$$\sum_{i \in s^p} \rho_{\kappa(i)}(y^p, \mathcal{K}(\kappa, s^p)) - f(y^p, \mathcal{K}(\kappa, s^p)) = 0.$$

Condition (1) states that all agents must maximize their utilities over jurisdiction type, public goods levels, and crowding assignments. Condition (2) states that, given the price system, no firm can make positive profits by entering the market and offering any other jurisdiction with any other public goods levels. Condition (3) requires that equilibrium jurisdictions exactly cover their costs.

A Tiebout admission price equilibrium is, in most respects, a standard competitive equilibrium notion. Given the prices, consumers are assumed to maximize utility and firms to maximize profit. Just as we expect from a price-taking equilibrium for private goods, prices are anonymous in that the prices consumers and firms pay do not depend on any unobservable traits. In this respect, the Tiebout admission prices are very different from the personalized prices of a Lindahl equilibrium concept. One undesirable feature of Tiebout admission prices, however, is that they require that jurisdictions with each possible crowding profile and every possible public goods level have a separate price. Generally this requires an infinite number of prices. Thus far, attempts to reduce the dimensionality of the price space have come at the expense of generality: core equivalence results with alternative price mechanisms are typically only attainable for a small class of economies. Those results are presented below. We now state core equivalence theorems for the Tiebout admission price equilibrium. Theorem 3 states that all Tiebout admission price equilibrium states are also core states. An immediate corollary of Theorem 3 is a first welfare theorem.

Theorem 3. *If the state $(X, Y, \kappa, S) \in F$ and the price system ρ constitute a Tiebout admission price equilibrium, then (X, Y, κ, S) is a core state.*

Theorem 4 shows that every core state can be decentralized as a Tiebout admissions price equilibrium for some Tiebout admission price system. Theorems 3 and 4 combine to prove that the set of equilibrium states of the economy is equivalent to the core. Proofs of both results are provided in Conley and Wooders (2001).

Theorem 4. *If an economy satisfies SSGE and for all $t \in \mathcal{T}$, u_t is continuous in x, then for each state (X, Y, κ, S) in the core, there exists a price system ρ such that ρ and (X, Y, κ, S) constitute a Tiebout equilibrium.*

As stated above, each of the various models of crowding are special cases of the differential genetic types and endogenous crowding types model. As a result, the above core equivalence result shows that anonymous decentralization is possible in each of these sub-models. Note, however, "anonymous" when applied to the *differentiated* crowding model loses it economic relevance. Although prices formally depend only on crowding types, in this special subcase crowding types and taste types are exogenous and perfectly correlated. Thus, prices in effect depend upon taste and so are not truly anonymous.

To address the issues of non-linearity and infinite dimensionality of admission prices, alternative price structures have been explored. The most prominent alternative price structure is Lindahl prices. A Lindahl price system in the context of this local public goods model specifies two prices: a per-unit price list for the public goods production levels and a participation price. A Lindahl price system is anonymous if those prices are independent of agents' tastes. For the case of anonymous crowding, Wooders (1978) showed the first anonymous core decentralization result for Lindahl prices.[5] (For this and all of the alternative price structures, all of the core equivalence results require some kind of small group effectiveness assumption). Notably, however, the Wooders (1978) result is restricted to an economy with constant-returns-to-scale production technology. Boadway (1980), Berglas and Pines (1980), Scotchmer and Wooders (1987), Barham and Wooders (1998), and many others confirm this result. In the case of differentiated crowding, nonanonymous Lindahl price equilibrium states and the core are equivalent but anonymous Lindahl price equilibrium states are generally strictly contained in the core (see Conley and Wooders (1998), Scotchmer and Wooders (1986), and Wooders (1981)). In the crowding types framework, the core is equivalent to the set of nonanonymous Lindahl price equilibria and is generally larger than the set of anonymous Lindahl price equilibria (see Conley and Wooders (1998)). Thus, although Lindahl prices offer a more appealing price system in that they are finite dimensional, the only anonymous core decentralization results seem to be attainable for the small class of economies in which production technology is linear.

Another pricing system, finite cost shares, addresses the infinite dimensionality of admission prices but with slightly less structure than Lindahl prices. Finite cost shares

[5] In Pauly (1972), agents are essentially identical and hence "anonymous" decentralization trivially holds.

specify for each crowding type and for each jurisdiction two prices: a list of shares of the cost of providing any levels of public goods, and a jurisdiction participation price. This slight restriction on the structure on prices allows an anonymous core decentralization result in the anonymous crowding framework without requiring linear technology. Conley and Smith (2002) show that if the cost of producing the public good is increasing (but not necessarily linear), the core and the set of anonymous finite cost share equilibrium states are equivalent when crowding is anonymous. However, this result does not hold for more than one crowding type and the generalization comes at a price: a finite cost share system requires that agents fully know the cost structure for public goods production.

4.1 Existence

One problem with the core and equilibrium approach to coalition economies is that, in general, the core may be empty. Pauly (1970) shows and Wooders (1978) confirms that even large and very nice economies generally have empty cores. The problem arises because most local public goods economies are characterized by jurisdictional structures which are optimal in that they maximize per-capita utility. But, unless the right number of each type of individual just happens to occur in the population, the population cannot sort itself into these optimal jurisdictional structures. For examples of this phenomenon, see Conley and Wooders (1997b). It should also be noted that this is not simply an integer problem; it occurs because the types need to occur in the correct proportion even if fractional agents were allowed.

The nonexistence remains a significant theoretical issue. However, the theoretical issue may not be of very much real world importance because, although the *exact* core may fail to exist, the *approximate core* will in general exist for large economies. In the context of local public economies, the approximate core has a natural interpretation. Basically we need only require that potential blocking coalitions to pay a small cost of defecting. It can be shown that the core exists for arbitrarily small jurisdiction formation costs (for a sufficiently large population). Various formulations of the approximate core and the corresponding existence results may be found in Wooders (1980), Kaneko and Wooders (1982), Shubik and Wooders (1983), and Wooders (1983). Alternatively, although the exact core may not exist in finite economies, several recent papers show that it will exist in continuum versions. For examples of the continuum approach, see Hammond, Kaneko, and Wooders (1989), Cole and Prescott (1997), Conley and Wooders (1997b), and Ellickson *et al.* (1999).

4.2 Non-cooperative Solutions

The above discussion of existence issues suggests that the core as a solution concept may be too strong. Under the core approach, coalitions have control over who is admitted to their membership. For example, firms, academic departments, and condominium cooperatives control admittance to their clubs. Furthermore, coordinated moves such as coalitional defections are often considered. Under a Nash approach, individual agents are freely mobile in that coalitions must admit anyone who applies (and pays the necessary price); that is, our society institutionally enforces an individual's right to freely migrate between jurisdictions. However, Konishi, Le Breton and Weber (1998) show that the Nash equilibrium approach is generally too weak. When it exists, the Nash equilibrium in of a finite local public goods economy is often not efficient.[6] Showing the existence of equilibria in the Nash-based approaches often runs into the same problems we find the core approach: the population size (and proportion of types) must be just right.[7]

Conley and Konishi (2002) offer a refinement of Nash equilibrium for local public goods economies called the *migration-proof Tiebout equilibrium.* In this approach, both

[6] This approach was first pursued by Kalai, Postlewaite, and Roberts (1979) and Guesnerie and Oddou (1979, 1981) with additional work by Konishi, Le Breton, and Weber (1997). See Konishi (1996) and Nechyba (1997) for a survey of work in this area.

[7] See Pauly (1970) and Wooders (1978).

coalitional and unilateral deviations are permitted but all deviations must be credible in the sense that they are stable against new migration. That is, deviations are credible if none of the agents who were left behind would want to follow the deviators to the new coalition. Conley and Konishi show that the exact migration-proof Tiebout equilibrium exists and is unique in large finite economies. Furthermore, they find that these equilibria are asymptotically efficient. However, this has only been proven for a small class of economies (identical agents with single-peaked preferences) and how far this approach can be extended is unclear.

4.3 Other Results and Future Research

An interesting question in these coalitional economies and an issue which has been important to the Tiebout literature is how agents sort themselves by tastes. If optimal jurisdictions are taste homogeneous, much can be concluded about efficient organization of our social and economic institutions. For example, the best way to educate college students would be in schools of like-minded students such as the London School of Economics and the Juilliard School of the Arts. Wooders (1978) shows that when crowding is anonymous, optimal jurisdictions will be taste homogeneous. When crowding is differentiated and when crowding types are exogenous, optimal jurisdictions will generally be taste heterogeneous. See Wooders (1989), Wooders (1997), Conley and Wooders (1995), and Conley and Wooders (1997a) for these results. If crowding types are endogenous and agents have the same genetic abilities, Conley and Wooders (1996) show that optimal jurisdictions will be taste homogeneous. Finally, if crowding types are endogenous and agents have different genetic abilities, equilibrium jurisdictions will generally be taste integrated.

The literature discussed herein has focused primarily on single-membership jurisdictions. However, we could also consider a “club” approach in which agents are allowed to be members of multiple clubs which provide different public goods. Buchanan (1965) first introduced the idea of a club good and the club approach to Tiebout’s hypothesis comprises a large part of the public economics literature. If agents are allowed to join multiple clubs, several interesting modeling questions arise: Should an agent's preferences depend directly on the intensity of his own and others' club usage? Or should agents be endowed with a finite amount of time which they may allocate across clubs? And to what extent will clubs specialize in a particular service rather that offering a spread of public goods?

Although these local public goods models are based on locational choice in the sense that each agent is choosing a jurisdiction in which to reside, land allocation has not been explicitly modeled. Of course, land allocation has been extensively explored in urban economics and regional science, but these studies do not generally treat the issues the manner of studying game theoretic coalitions. There are likely to be many interesting spillovers between the two approaches.

References

Arnott, R. and M. Kraus (1998): "When are anonymous congestion charges consistent with marginal cost pricing?," *Journal of Public Economics,* **67**, 45-64.

Barham, V. and M. H. Wooders (1998): "First and second welfare theorems for economies with collective goods," in *Topics in Pubic Finance,* by (D. Pines, E. Sadka, and I. Zilcha, Eds.), Cambridge University Press.

de Bartolome, C. (1990): "Equilibrium and efficiency in a community model with peer-group effects," *Journal of Political Economy,* **98**, 110-133.

Benabou, R. (1993): "Workings of a city: location, education, and production," *Quarterly Journal of Economics,* **105**, 619-652.

Berglas, E. (1976): "Distribution of tastes and skills and the provision of local public goods," *Journal of Public Economics,* **6**, 409-423.

Berglas, E., E. Helpman and D. Pines (1982): "The economic theory of clubs: Some clarifications," *Economics Letters,* **10**, 343-8.

Berglas, E. and D. Pines (1980): "Clubs, as a case of competitive industry with goods as a variable quality," *Economics Letters,* **5**, 363-366.

Berglas, E. and D. Pines (1981): "Clubs, local public goods and transportation modes; a synthesis," *Journal of Public Economics,* **15**, 141-162.

Bewley, T. (1981): "A critique of Tiebout theory of local public expenditure," *Econometrica,* **49**, 713-740.

Boadway, R. (1980): "A note on the market provision of club goods," *Journal of Public Economics,* **13**, 131-137.

Brueckner, J. (1994): "Tastes, skills and local public goods," *Journal of Urban economics,* **35**, 201-220.

Buchanan, J. M. (1965): "An economic theory of clubs," *Economica,* **32**, 1-14.

Cole, H. and E. Prescott (1997): "Valuation equilibria with clubs," *Journal of Economic Theory,* **74**, 19-39.

Conley, J. and H. Konishi (2002): "Migration-proof Tiebout equilibrium: existence and asymptotic efficiency," *Journal of Public Economics,* **86**, 243-262.

Conley, J. and S. C. Smith (2003): "Finite decentralization in a Tiebout economy with crowding types," *Social Choice and Welfare,* **20**, 49-75.

Conley, J. and M. H. Wooders (1995): "Hedonic independence and taste-homogeneity of optimal jurisdictions in the core of a Tiebout economy with crowding types," *University of Illinois Office of Research Working Paper,* pp. 95-118.

Conley, J. and M. H. Wooders (1996): "Taste-homogeneity of optimal jurisdictions in a Tiebout economy with crowding types and endogenous educational investment choices," *Recherche Economiche,* **50**, 367-87.

Conley, J. and M. H. Wooders (1997a): "Equivalence of the core and competitive equilibrium in a Tiebout economy with crowding Types," *Journal of Urban Economics,* **41**, 421-440.

Conley, J. and M. H. Wooders (1997b): "Anonymous pricing in public goods economies," in *Topics in Pubic Finance,* by (D. Pines, E. Sadka, and I. Zilcha, Eds.), Cambridge University Press.

Conley, J. and M. H. Wooders (1998): "Anonymous Lindahl pricing in a Tiebout economy with crowding types," *Canadian Journal of Economics,* **31**, 952-74.

Conley, J. and M. H. Wooders (2001): "Tiebout economies with differential genetic types and endogenously chosen crowding characteristics," *Journal of Economic Theory,* **98**, 261-94.

Cornes, R. and T. Sandler (1996) (The theory of externalities, public goods and club goods): *Cambridge University Press.* Second Edition.

Davis, O.A. and A.B. Whinston (1967): "On the distinction between public and private goods," *American Economic Review,* **57**, 360-73.

DeSerpa, A.C. (1977): "A theory of discriminatory clubs," *Scottish Journal of Political Economy,* **24**, 33-41.

DeVany, A.S. and T.R. Saving (1977): "Product quality, uncertainty, and regulation: The trucking industry," *American Economic Review,* **67**, 583-94.

Ellickson, B. (1979): "Local public goods and the market of neighborhoods," in *The Economics of Neighborhoods,* by D. Segal, editor, Academic Press (New York), 263-94.

Ellickson, B. and B. Grodal, S. Scotchmer, and W. Zame (1999): "Clubs and the market," *Econometrica,* **67** 1185-1218.

Guesnerie, R. and C. Oddou (1979): "On economic games which are not necessarily superadditiitve," *Economic Letters,* **3**, 301-306.

Guesnerie, R. and C. Oddou (1981): "Second best taxation as a game," *Journal of Economic Theory,* **25**, 67-91.

Hammond, M., and M. Kaneko and M. H. Wooders (1989): "Continuum economies with finite coalitions: core, equilibria and widespread externalities," *Journal of Economic Theory,* **49**, 113-134.

Hillman, A.L. and P.L. Swan (1979): "Club participation under uncertainty," *Economics Letters,* **4**, 307-12.

Kalai, E., and A. Postlewaite and J. Roberts (1970): "A group incentive compatible mechanism yielding core allocations," *Journal of Economic Theory,* **20**, 13-22.

Kamien, M.I., N.L. Schwartz, and D.J. Roberts (1973): "Exclusion, externalities, and public goods," *Journal of Public Economics,* **2**, 217-30.

Kaneko, M. and Wooders, M. H. (1982): "Cores of partitioning games," *Mathematical Social Sciences,* **3**, 313-327.

Knight, F.H. (1924): "Some fallacies in the interpretation of social cost," *Quarterly Journal of Economics,* **38**, 582-606.

Konishi, H. (1996): "Voting with ballots and feet: Existence of equilibrium in a local public good economy," *Journal of Economic Theory,* **68**, 480-509.

Konishi, H. (1997): "Free mobility equilibrium in a local public goods economy with congestion," *Research in Economics,* **51**, 19-30.

Konishi, H., M. Le Breton, and S. Weber (1998): "Equilibria in finite local public goods economies," *Journal of Economic Theory,* **79**, 224-244.

McGuire, M. (1974): "Group size, group homogeneity, and the aggregate provision of a pure public good under Cournot behavior," *Public Choice,* **18**, 107-26.

McGuire, M. (1991): "Group composition, collective consumption, and collaborative production," *American Economic Review,* **81** 1391-1407.

Millward, R (1970): "Exclusion costs, external economies, and market failure," *Oxford Economic Papers,* **22**, 24-38.

Nechyba, T. (1997): "Existence of equilibrium and stratification in local and hierarchical Tiebout economies with property taxes and voting," *Economic Theory,* **10**, 277-304.

Nichols, D., E. Smolensky, and T.N. Tideman (1971): "Discrimination by waiting time in merit goods," *American Economic Review,* **61**, 312-23.

Ng, Y.K. and R.D. Tollison (1974): "A note on consumption sharing and non-exclusion rules," *Economic,* **41**, 446-50.

Oakland, W.H. (1972): "Congestion, public goods, and welfare," *Journal of Public Economics,* **1**, 339-57.

Pauly, M. (1967): "Clubs, commonality, and the core: An integration of game theory and the theory of public goods," *Economica,* **34**, 314-24.

Pauly, M. (1970): "Cores and clubs," *Public Choice,* **9**, 53-65.

Pigou, A.C. (1920) (The economics of welfare): *Macmillan.* London.

Samuelson, P (1954): "The pure theory of public expenditures," *Review of Economics and Statistics,* **36**, pp. 387-389.

Sandler, T. and J. Tschirhart (1980): "The economic theory of clubs: an evaluative survey," *Journal of Economic Literature,* **18**, 1481-15211.

Schwab, R. and W. Oates (1991): "Community composition and the provision of local public goods: a normalize analysis," *Journal of Public Economics,* **44**, 217-237.

Scotchmer, S. (1994): "Public goods and the invisible hand," in *Modern Public Finance,* by (J.M. Quickley and E. Smolensky, Eds.), pp. 93-125, Harvard University Press.

Scotchmer, S. and M. H. Wooders (1986): "Optimal and Equilibrium groups," *Harvard Discussion Paper 1251, .*

Scotchmer, S. and M. H. Wooders (1987): "Competitive equilibria and the core in economies with anonymous crowding," *Journal of Public Economics,* **34**, 159-174.

Shubik, M. and Wooders, M. H. (1983): "Approximate cores of replica games and economies: Part I. Replica games, externalities, and approximate cores," *Mathematical Social Sciences,* **6**, 27-48.

Shubik, M. and Wooders, M. H. (1983): "Approximate cores of replica games and economies: Part II. Set-up costs and firm formation in coalition production economies," *Mathematical Social Sciences,* **6**, 285-306.

Tiebout, C. (1956): "A pure theory of local expenditures," *Journal of Political Economy,* **64**, 416-424.

Tollison, R. D. (1972): "Consumption sharing and non-exclusion rules," *Economica,* **39**, 279-91.

Westhoff, F. (1977): "Existence of equilibria in economies with a local public good," *Journal of Economic Theory,* **14**, 84-112.

Wooders, M. H. (1978): "Equilibria, the core, and jurisdiction structures in economies with a local public good," *Journal of Economic Theory,* **18**, 328-348.

Wooders, M. H. (1980): "The Tiebout hypothesis: Near optimality in local public good economies," *Econometrica,* **48**, 1467-1486.

Wooders, M. H. (1981): "A limit theorem on the ϵ-core of an economy with public goods," *,* National Tax Institute of Japan, Paper No. 20.

Wooders, M. H. (1983): "The epsilon-core of a large replica game," *Journal of Mathematical Economics,* **11**, 277-300.

Wooders, M. H. (1989): "A Tiebout theorem," *Mathematical Social Sciences,* **18**, 33-55.

Wooders, M. H. (1992): "Inessentiality of Large Groups and the Approximate Core Property; An equivalence Theorem," *Economic Theory,* **2**, 129-147.

Wooders, M. H. (1997): "Equivalence of Lindahl equilibrium with participation prices and the core," *Economic Theory,* **9**, 115-127.