Private Benefits, Warm Glow and Reputation in the†
Free and Open Source Software Production Model

John P. Conley*

and

Fan-Chin Kung**

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* Department of Economics, Vanderbilt University, j.p.conley@vanderbilt.edu

** Department of Economics and Finance, City University of Hong Kong and Department of Economics, East Carolina University, kungfc@cityu.edu.hk

Abstract

A great deal of production and consumption behavior takes place in the context of social organizations that seem to fall outside of the traditional paradigm of profit/utility maximization. These organizations are voluntary in nature and rely on contributions from Members to achieve their objectives. Examples include the Linux operating system and other FOSS projects, political movements, churches and religious groups, Habitat for Humanity and similar charitable organizations. In this paper, we consider a world containing agents with heterogeneous abilities who may voluntarily choose to make effort contributions to one or more different public projects. Agents are motivated by a desire to be seen as significant contributors to important and valuable projects, the warm glow from the act of contributing, and a desire to directly enjoy the benefits of the project when complete. We show that Nash equilibria exist and study how the parameters of the model affect the equilibrium outcomes.

1. Introduction

The free and open source software (FOSS) movement is growing increasingly important to how we organize our technical lives. More than half the webservers use the free open source product Apache, and Linux has a greater than 10% share of the server market and about 4% of users' desktops.[1] The essential feature that distinguishes open source from commercial software is that the source code is "open" in the sense that users and developers are allowed to see the human readable code as opposed to having access only to the machine readable compiled code. This in turn makes it possible for users to make modifications, fix bugs, and offer extensions of the code. Perhaps as a consequence, a culture has arisen around FOSS in which the software is written and maintained by groups of volunteer programmers and then the software is provided free of charge to the users. See Eric Raymond's (1999) excellent book on the history of the FOSS movement for more details.

Of course, this calls the obvious question: why do skilled programmers voluntarily donate significant amounts of effort to such projects? This issue has generated large experimental, empirical and theoretical literatures in Law, Sociology, Economics and many other fields. We will not attempt a survey here (see Rossi 2004). Our reading of this literature is that the major motivations for these voluntary contributions boil down to the following (1) a desire to consume the public good produced as a result of these contributions, (2) an expectation of reaping benefits from being seen to contribute, and (3) taking joy in act of contributing itself. We elaborate briefly on all three below.

Bergstrom, Bloom, Varian(1986) and Warr(1983) provide the classical theoretical models of voluntary contributions motivated solely by the desire to consume the resulting public good. They consider the Nash equilibria of games in which agents choose to make contributions to the point that their private marginal benefits equal their private marginal costs, taking the contributions of other agents as given. While contribution levels are positive, free riding tends to become significant when group size becomes

[1] See www.w3schools.com/browsers/browsers_ os.asp.

large.

At first take, it might seem that this contradicts the empirical evidence of high contribution levels by large numbers of agents in the FOSS world. However, using survey data, Lakhani and Wolf (2003) and Hertel *et al.* (2003) find instead that a significant number of FOSS contributors in fact credit their efforts to the expectation of personally benefiting from the extensions that they add to a project. Of course, this is consistent with the voluntary contribution story, but also suggests something that is quite distinctive about FOSS projects as compared to public goods in general. Specifically: the contributions to FOSS projects are fundamentally inhomogeneous in nature (as, for example, money or volunteered time contributed standard public good projects might be). One user may want a piece of software ported to a specific operating system, another may want it translated into Finnish and yet another may want to be able to extract data into an Excel spreadsheet. While these contributions are purely pubic and generate non-rival benefits, a user with a need for a specific addition might be better off making the contribution himself than free riding in hopes that another user with an identical need will eventually happen along and make the contribution for him. Thus, the levels of free-riding are likely to be much lower than would be seen in more typical voluntary contribution situations.

The idea that FOSS public projects have this kind of modularity has led to an interesting literature in itself. Bitzer and Schroder (2002) consider a model in which modular contributions must take place in discreet packets and are made by agents with different abilities. Yildirim (2006) considers a model of dynamic voluntary contributions to a public project broken down into a finite set of modular subprojects. This is a nice effort to get at the special features of the FOSS projects that make the voluntary contribution model more empirically relevant than in most other public contexts. Yanase (2006) also considers a dynamic voluntary contribution model, but focuses on subsidy schemes that generate optimal steady states. In short, there are both theoretical and empirical reasons to believe that voluntary contributions are a significant real world factor in getting FOSS projects going.[2]

Another very frequently discussed motivation to make voluntary contributions is a desire to gain reputation. In well cited papers, Learner and Tirole (2002, 2005) give a general discussion of what motivates FOSS contributors, but focus on the idea that contributors are trying to signal their ability which in turn leads to future benefits such as better pay and better jobs. Spiegel (2005) provides a nice formal analysis of this phenomenon in a FOSS signaling game. In a more general context, Rege and Telle (2002) and Rege (2004) discuss how reputation, status and social norms play a role in directly enforcing voluntary contributions. Thus, we can imagine that the "reputational" motivation to make voluntary contributions may be due to a pull from the prospect of getting better jobs by signaling an agent's ability, or a push from the fear of being criticized or ostracized for failing to make such contributions.[3]

Exactly how one attains reputation is an interesting question and many things might influence an agent here. For example, it might matter how many others are making contributions. On the one hand, with too few contributors, the project will never take off and be of significance. Contributions in this case will be to a dead-end project and coders will not gain reputation. On the other hand, with too many contributors, the contributions of any one coder will be lost in the sea of other contributions. Again, not much reputation will be gained. The underlying objective value of the project will also affect the decision to make contributions as will the ability that a given coder has to make contributions to a given project. We will attempt to capture these considerations in the formal model we develop below.

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[2] Also note that many other forms of voluntary contribution games have been considered. See, for example, Bagnoli and Lipman (1989) who implement the core using a voluntary contribution mechanism, Koster, Riejnierse and Voorneveld (2003) who consider the strong Nash equilibria of a voluntary contribution game with multiple public projects and Laussel and Palfrey (2003) for a recent example of a voluntary contribution mechanism that achieves efficiency.

[3] In the context of fundraising, Kumru and Vesterlund (2008) show theoretical and experimental results that indicate the importance of status in encouraging contributions by both high and low status individuals. Here, status allows one to be a leader and influence the actions of others. Also see Guth, *et al.* (2007) and Poggrebna *et al.* (2008) for additional studies that relate reputation, status and leadership in voluntary contribution settings and Quint and Shubik (2001) for a more general treatment of games of status.

We also find support in the literature for more altruistic and indirect motivations for contributions in the FOSS world. Lakhani and Wolf (2003) find that many contributors cite "intrinsic" motivations for their contributions (consumption and reputation, in contrast, are classed as "extrinsic" motivations). Software engineers contribute code to FOSS projects simply because they enjoy writing code; they take pleasure in the act of production itself. In economic terms, this is a manifestation of "warm glow". See Andreoni (1990, 1995) for the canonical treatments of this in economics. The idea is that the act of contributing is its own reward. One is not influenced by the desire to consume the public good itself or by the level of contributions others make. Thus, warm glow might come from the happy knowledge that one is the author of a good work (a more altruistic motivation), or it might come from pleasure in the act of production itself (an indirect, but more selfish motivation). The latter seems to be particularly important in the open source world. Writing clean and elegant code is an art-form, and coders take pleasure in exercising their skills.

While the motivating example we consider is the FOSS movement, the incentives discussed above clearly are not unique to this environment. People contribute to social service organizations, political advocacy groups, neighborhood watches, academic journals and intellectual movements, just to name a few examples, for essentially the same reasons. Indeed the volunteerism is enormous. A recent estimate for 2001 by the *Independent Sector* [4] suggests that 89 percent of US households make contributions and that the average annual contribution for contributors is $1,620. In addition, 83.9 million American adults volunteer the equivalent of over 9 million full-time employees at a value of $239 billion. Thus, the question of how agents choose to allocate effort between private consumption and the large potential array of public projects is of great empirical importance.

With this in mind, our objective in this paper is to develop a model with many agents and many public projects. Agents have different abilities and are better or worse

[4] *Independent Sector* is "the leadership forum for charities, foundations, and corporate giving programs committed to advancing the common good in America and around the world". See http://www.independentsector.org/programs/research/gv01main.html

at converting effort into contributions to each specific project. Agents have utility functions that parametrically mix the Personal Benefit, Warm Glow and Reputation motivations for contributions. Thus, we combine all the most significant motivations discussed in the literature in a simple, but general model with an arbitrary number of agents and projects. As far as we are aware, this mixing of motivations is a novel feature.

We show that Nash equilibria exists in general. We distinguish between donors who are small and large in equilibrium (that is, donors who contribute less than or more than half of the total contributions made to a project in equilibrium, respectively). We find that contributions are strategic complements for small donors motivated mainly by Personal Benefits, and large donors motivated by Reputation, and are strategic substitutes for small donors motivated mainly by Reputation, and large donors motivated by Personal Benefits.

We also carry out a number of comparative static exercises for two special cases of the model: a two person economy (a marriage for example) and a large symmetric economy in which all agents have equal abilities. We find several expected results. For example, in most cases, increasing the ability of an agent increases his contributions to a given project. However, unexpectedly, in the case of small donors motivated by Reputation, increasing ability can actually drive contributions down for the case of symmetric agents. Increasing the objective value of a project has the expected effect of raising contributions of small donors motivated by Personal Benefits and large donors motivated by Reputation in a two person economy but in other cases, the effect can go either way.

What this shows is that even in a relatively simple economy, the motivations of agents to make voluntary contributions interact in a complex way. Intuitions often lead us astray. While the good news is that agents will come to an equilibrium in this game, the array of voluntary contributions form a complicated pattern. Our hope is that this study will go some way in exposing and explaining how these decisions are made when agents interact strategically and have many choices about where to contribute their efforts.

The plan of the paper is as follows. In section 2, we develop the model. In section 3, we show equilibria exists. In section 4, we show when contributions of agents are strategic complements and substitutes. In sections 5, we give comparative statics for the two person and the symmetric cases. In section 6, we give a matrix that summarizes these results. Section 6 concludes.

2. The Model

We consider an economy with I agents and N projects open for voluntary contributions. Each individual has a vector of basic abilities $a_i \in \Re^n$ where a_i^n is taken as an index of agent i's competence at contributing to project n. Given this, agents choose effort vector $(e_i) \in \Re^n$ where $e_i^n \in \Re_+$ is taken as the effort agent i devotes to project n. Expending effort generates disutility for the agent according to the function: $g_i\left(\sum_n e_i^n\right)$. We shall assume that $g_i' \geq 0$, $g_i'' \geq 0$ and $\lim_{x\to\infty} g'(x) > 0$.

The net contribution the project receives from this choice depends both on ability and effort:

$$c_i^n = a_i^n e_i^n$$

Each project n has an *objective impact parameter* o^n and the total social benefits received from a project are proportional to the product of the total contributions made to the project and the impact parameter: $o^n \sum_i c_i^n$. Each agent can claim a share of responsibility: $\frac{c_i^n}{c_i^n + \sum_{-i} c_j^n}$ of the project n.

We assume that agents care about both the objective value and his share of responsibility. More specifically, we assume the benefits agents get from project is additively separable and follows the following Cobb-Douglas form:

$$v_i^n\left(c_i, c_{-i}\right) = \left(\frac{c_i^n}{c_i^n + \sum_{-i} c_j^n}\right)^{\theta} \left(o^n \left(c_i^n + \sum_{-i} c_j^n\right)\right)^{1-\theta} = \left(o^n\right)^{1-\theta} \left(c_i^n\right)^{\theta} \left(c_i^n + \sum_{-i} c_j^n\right)^{1-2\theta}.$$

where $\theta \in [0, 1]$ measures the degree in which agent i cares about responsibility. Putting this together with the cost of effort gives the following utility function:

$$u_i(c_i) = \sum_{n=1}^{N} v_i^n(c_i, c_{-i}) - g_i\left(\sum_n \frac{c_i^n}{a_i^n}\right)$$

Note that this model reflects well established approaches to voluntary contribution in the existing literature as special cases.

$\theta = 0$ This is a form of the Bergstrom-Bloom-Varian/Warr approach where agents make contributions only because they value the public good that results. Of course our model is not a generalization of this approach in a formal sense both because we use a specific Cobb-Douglas functional form and because the impact parameter (o^n) implies a degree of commonality in how agents view the benefits coming from projects.

$\theta = \frac{1}{2}$ This leads to an Andreoni type model of pure Warm Glow. All cross partials are zero and so agents care only about their own contributions to projects in proportion to how objectively valuable the projects are. Thus, agents are motivated by the good feeling they get from contributing to valuable projects (perhaps because giving feels good, or because the act of production is itself enjoyable if other people appreciate it). They don't care at all how much other agents contribute.

$\theta = 1$ This leads to a pure responsibility model. Agents care about the community seeing that they have made significant contributions to projects. Agents don't particularly care how valuable the projects happen to be. They simply want to be seen to have made a proportionally large contribution.

It will turn out the behavior of agents will depend very heavily on which of these three motivations dominates. We find it convenient to consider three separate cases:

- **Personal Benefit Case**: $0 < \theta < 1/2$. Agents tend to be motivated to contribute more by the Personal Benefits they get from consuming the correspondingly higher levels of public good that their own contributions produce than a desire to receive credit for these contributions. Of course, agents are still partially motivated by Warm Glow. We will see below that the influence of Warn Glow is much more significant for agents who make large donations than small donations.

- **Pure Warm Glow Case:** $\theta = 1/2$ Agents are motivated purely by the joy of giving to useful projects. They do not care what others contribute either for the effect it might have on the share of the credit they receive for the project or the overall quantity of the project available for them to consume personally.
- **Reputation Case:** $1/2 < \theta < 1$. Agents tend to be motivated to contribute by the share of the credit they will be awarded to building valuable projects. This might be simply because it improves their social status, or because indirect rewards (better jobs, promotion tenure, etc) are more likely to be awarded to those with higher status. Modeling the details of the reward structure and how this affects the incentive to acquire Reputation is interesting in itself and should be explored in future research. In a symmetric way to the first case, agents are also partially motivated by Warm Glow. We will see below that the influence of Warm Glow is much more significant for agents who make small donations than large donations.

3. Existence and the General Case

In this section we lay the foundations for analyzing the nature of the equilibrium. The most natural approach is to explicitly solve for the Nash equilibrium in the contribution game. This would require finding a solution to the following system of first order conditions:[5]

[5] Note that in the interest of simplicity, we treat contribution rather than effort as a choice variable. This is without loss of generality since there is a fixed linear relationship between the two via the ability coefficients.

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0 \text{ for all } i \text{ and all } n.$$

Unfortunately, finding a closed form solution does not seem to be possible even with fully specified g_i functions. This is mainly because effort cost, $g()$, is not separable across projects. As a result, the contribution levels to each project and also the ability coefficients of an agent for each project are arguments in each of these first order conditions through $g_i'()$.

Since no general solution is available, our strategy will be to consider a series of special cases in the following sections. All of these will rely on interpreting the first-order and second-order derivatives of v_i^n with respect to c_i^n, c_{-i}^n and o_i^n.[6]

Derivatives of the Value Function

$$\frac{\partial v_i^n}{\partial c_i^n} = (o^n)^{1-\theta} (c_i^n)^{\theta-1} \left(c_i^n + c_{-i}^n\right)^{-2\theta} \left(\theta c_{-i}^n + (1-\theta) c_i^n\right) > 0.$$

$$\frac{\partial^2 v_i^n}{(\partial c_i^n)^2} = (o^n)^{1-\theta} (c_i^n)^{\theta-2} \left(c_i^n + c_{-i}^n\right)^{-1-2\theta} \theta \left[(\theta-1)\left(c_{-i}^n\right)^2 + (\theta-1)(c_i^n)^2 - 2\theta c_i^n c_{-i}^n\right] < 0.$$

$$\frac{\partial v_i^n}{\partial c_j^n} = \frac{\partial v_i^n}{\partial c_{-i}^n} = (o^n)^{1-\theta} (1-2\theta) (c_i^n)^{\theta} \left(c_i^n + c_{-i}^n\right)^{-2\theta}$$

$$\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n} = \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_{-i}^n} = (o^n)^{1-\theta} (1-2\theta) (c_i^n)^{\theta-1} \left(c_i^n + c_{-i}^n\right)^{-1-2\theta} \theta \left[c_{-i}^n - c_i^n\right]$$

$$\frac{\partial^2 v_i^n}{\partial c_j^n \partial c_k^n} = \frac{\partial^2 v_i^n}{\left(\partial c_{-i}^n\right)^2} = (o^n)^{1-\theta} (1-2\theta)(-2\theta) (c_i^n)^{\theta} \left(c_i^n + c_{-i}^n\right)^{-1-2\theta}$$

$$\frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n} = (1-\theta) (o^n)^{-\theta} (c_i^n)^{\theta-1} \left(c_i^n + c_{-i}^n\right)^{-2\theta} \left(\theta c_{-i}^n + (1-\theta) c_i^n\right) > 0$$

While several of these derivatives can be unambiguously signed, others are not signable in general. It turns out that the signs alternate depending on two factors: (i)

[6] To simplify notation, let $c_{-i}^n = \sum_{j \neq i} c_j^n$. Note also that the first equations also implies that $\lim_{c_i^n \to 0} \frac{\partial v_i^n}{\partial c_i^n} = \infty$.

whether i's total share of contributions to a given project are greater than half **(Large donor case)** or less than half **(Small donor case)** and (ii) the size of θ, that is, whether players care more about his own share of responsibility or the objective value of a project. The table below gives the details.

Signs of the Value Function's Derivatives by Case			
	Personal Benefit Case	Warm Glow Case	Responsibility Case
$\frac{\partial v_i^n}{\partial c_j^n}$	> 0	$= 0$	< 0
$\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}$	small donor >0 large donor <0	$= 0$	small donor <0 large donor >0

This leads to some interesting economic interpretations. The first derivative $\frac{\partial v_i^n}{\partial c_j^n}$ is the effect of the other agent's contributions on a given agent's welfare. If agents are mainly motivated by wanting to consume the project, then other agents' contributions are beneficial. If agents are mainly motivated by wanting credit, then other agents' contributions are harmful.

The second derivative $\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}$ is the effect of other agent's contributions on a given agent's marginal benefit of contributions. To understand this, note that as a donor increases his total share of contributions, it becomes increasingly costly for him to increase his own share more. For example, suppose I contribute nine out of ten total units contributed to a project. By contributing an extra unit, I can increase my share of responsibility from 90% to 91% (now 10 out of 11 total units). On the other hand, if I am contributing nine out of eighteen units to begin with, contributing one extra unit causes my responsibility share to go from 50% to 52.6%. Thus, if other agents increase their contributions by eight units, my own contributions become more effective at increasing my own share of responsibility for a project if I am a large donor to begin with. The opposite is true if I am a small donor. Given this, we have four cases. These cases are the key to understanding most of the results in the subsequent sections.

For the **large donor/Reputation case**, this derivative is positive. This says if the small donors increase contributions, it becomes less costly for the large donor to increase his share, thus the MB of his contributions goes up.

For the **small donor/Reputation case**, this derivative is negative. This says that if other donors increase contributions, it becomes more costly for a small donor to increase his share, thus the MB of his contributions goes down.

For the **large donor/Personal Benefit case**, this derivative is negative. This says that if the other donors increase their contributions, there are two competing effects. First, the large donor gets more consumption implying the marginal benefit to consumption of his own contributions goes down. Second, the project is better supported and yields more benefits overall which raises the marginal Warm Glow the agent gets from increasing his contribution. (That is, agents gets more Warm Glow on the margin from contributing to better projects.) For Large donors, the first effect is larger and so the net MB of his contributions goes down.

For the **small donor/Personal Benefit case**, this derivative is positive. This says that if the other donors increase contributions, we see the same two competing effects as above. However, the Warm Glow effect is larger and so the MB of his own contributions go up.

For all donors in the Warm Glow case, the competing factors cancel out and this derivative is exactly zero. In the interest of space, we will not explicitly treat this non-generic boundary case below.

This system of derivatives allows us to show that equilibrium exists, in general.

Theorem 1. *A Nash equilibrium in contribution levels exists for all values of the model's parameters.*

Proof/

Examining the first order conditions we see

$$\lim_{c_i^n \to \infty} \frac{\partial v_i^n}{\partial c_i^n} = 0$$

Since $\lim_{x\to\infty} g'(x) > 0$ we conclude

$$\lim_{c_i \to \infty} \left(\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} \right) < 0,$$

This in turn implies that equilibrium contribution levels will be bounded. We can therefore restrict the strategy space of contributions to $c_i^n \in \left[0, \bar{B}\right] \equiv \mathcal{B}$ for all i and all n for some bounded $\bar{B} \in \Re$.

By construction $u_i(c)$ is continuous in c. It is also the case that $u_i(c)$ is strictly concave in c_i^n since $v_i^n(c)$ is strictly concave in c_i, and g_i is weakly convex in c_i.

Therefore, each agent i has a single-valued continuous best response function:

$$\bar{c}_i(c_{-i}) = \arg\max_{c_i} u_i(c).$$

Aggregating these together gives a single valued continuous mapping $\bar{c} = (\bar{c}_i(c_{-i}))_{i \in I}$ from $\mathcal{B}^{IN}$ to $\mathcal{B}^{IN}$ which is compact and convex. Existence of equilibrium now follows directly from Brower's fixed point theorem.

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4. Strategic Complements and Substitutes.

One factor that complicates our analysis is that the optimal contribution to a given project for an agent depends on how much he is contributing to other projects. These levels in turn depend on other agents' contributions, which depend on their vectors of ability parameters, the quality of the other projects and so on. Thus, all the parameters of the model affect the contribution decision of any given agent to any given project. For this section and the remainder of this paper, we therefore make the simplifying assumption that agents have *constant marginal disutility of effort* g_i'. This neatly breaks the linkage between projects. It implies that the optimal contribution to individual projects for each agent can be determined independently of the economic parameters and contribution levels for other projects. Thus, the first order condition for project n becomes:

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0 \text{ for all } i$$

This in turn defines player i's reaction function $\bar{c}_i^n\left(c_{-i}^n\right)$ taking c_{-i}^nas exogenous. Differentiating the first-order condition of player i with respect to c_j^n, we have:

$$\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2}\frac{\partial \bar{c}_i^n\left(c_{-i}^n\right)}{\partial c_j^n} + \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n} = 0,$$

and so:

$$\frac{\partial \bar{c}_i^n\left(c_{-i}^n\right)}{\partial c_j^n} = -\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}\Big/\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2}.$$

The question then is: are the contribution levels strategic complements $\left(\frac{\partial \bar{c}_i^n\left(c_{-i}^n\right)}{\partial c_j^n} > 0\right)$ or strategic substitutes $\left(\frac{\partial \bar{c}_i^n\left(c_{-i}^n\right)}{\partial c_j^n} < 0\right)$. The following theorem is very easy to show:

Theorem 2. *If the disutility of effort is constant then*

(i) Other agents' contribution levels are **Strategic Complements** *for a small donor motivated by Personal Benefits, and a large donor motivated by Reputation.*
(ii) Other agents' contribution levels are **Strategic Substitutes** *for a small donor motivated by Reputation, and a large donor motivated by Personal Benefits.*

Proof/

Since $\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2} < 0$, the cross partial derivative $\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}$ determines the sign of $\frac{\partial \bar{c}_i^n\left(c_{-i}^n\right)}{\partial c_j^n}$. The table in the previous section where these signs were determined for each of these cases, therefore, immediately implies the Theorem.

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Let us make a few remarks about this. First, consider small donors. For the Personal Benefits case, recall that it is actually Warm Glow that primarily motivates actions. Thus, the contributions of other agents improve the projects value and thus encourage small donors to increase their contributions. It is interesting to see a kind of virtuous circle in which agents who enjoy the act of contributing but who do not seek fame or credit reinforce each others' public spiritedness. On the other hand, when small donors care more about their reputations, contributions by other agents crowd out their own contributions. Thus, small donors seeking glory step on each others' toes. Second, for large donors, the opposite is true. When a large donor cares more about Personal Benefits, increased effort on the part of the remaining agents crowds out his contributions. Other agents provide the public good and this relieves the burden on him. On the other hand when the large donor cares about his Reputation, contributions by remaining agents cause him to add to his contributions. Again, this is because it increases the incremental reputational effect of adding contributions. Thus, large donors are spurred on by other donors in order to maintain the lion's share of the credit for a project.

5. Comparative Statics

In the last section, we examined the effects that agent's strategic choices had on one another. In this section we explore how equilibrium contribution levels respond to the underlying economic environment. In particular, we will attempt to show how the relative quality of projects and the ability of agents to contribute to them affect the equilibrium levels of effort and contribution chosen by an agent. We maintain the assumption of constant marginal cost of effort.

Begin by letting $\hat{c}_i^n(a^n, o^n)$ denote the equilibrium contribution of player i. To determine the impact of o^n on the agent's contributions, differentiate the first order conditions ($\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0$) with respect to o^n:

$$\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}\frac{\partial \hat{c}_i^n}{\partial o^n} + \sum_{j \neq i} \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}\frac{\partial \hat{c}_j^n}{\partial o^n} + \frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n} = 0 \text{ for all } i.$$

Clearly, $\frac{\partial \hat{c}_i^n}{\partial o^n}$ cannot be signed in general.

The impact of a_i^n on contributions from agent i and other agents j can be found by differentiating the first order conditions with respect to a_i^n:

$$\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}\frac{\partial \hat{c}_i^n}{\partial a_i^n} + \sum_{j \neq i} \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}\frac{\partial \hat{c}_j^n}{\partial a_i^n} = -\frac{g_i'}{(a_i^n)^2},$$

$$\frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2}\frac{\partial \hat{c}_j^n}{\partial a_i^n} + \sum_{k \neq j} \frac{\partial^2 v_j^n}{\partial c_j^n \partial c_k^n}\frac{\partial \hat{c}_k^n}{\partial a_i^n} = 0 \text{ for all } j \neq i.$$

Clearly, $\frac{\partial \hat{c}_i^n}{\partial a_i^n}$ and $\frac{\partial \hat{c}_j^n}{\partial a_i^n}$ are also unsignable in general.

To help understand what is going on, therefore, we will consider two special subcases: a two person game and a symmetric game.

5.1 Two-person Voluntary Contribution Game

Consider this as an analysis of how voluntary contributions are determined in marriages or partnerships. Note that in a two-person game, there must be one small and one large donor, at least generically. As a result, we can determine the comparative statics in detail. As before, the first-order conditions are as follows (with two players i and i).

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0,$$
$$\frac{\partial v_j^n}{\partial c_j^n} - \frac{g_j'}{a_j^n} = 0.$$

Theorem 3. *An increase in the project quality coefficient (o^n) will increase the contribution of the small donor if he is motivated by Personal Consumption and will increase the contribution of the large donor if he cares mainly about Reputation.*

Proof/

Differentiate the system of first order conditions

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0,$$

$$\frac{\partial v_j^n}{\partial c_j^n} - \frac{g_j'}{a_j^n} = 0.$$

with respect to o^n; we have

$$\begin{pmatrix} \frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2} & \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n} \\ \frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n} & \frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2} \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{c}_i^n}{\partial o^n} \\ \frac{\partial \hat{c}_j^n}{\partial o^n} \end{pmatrix} = \begin{pmatrix} -\frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n} \\ -\frac{\partial^2 v_j^n}{\partial c_j^n \partial o^n} \end{pmatrix}$$

When (i) $0 < \theta < 1/2$ and i is a small donor or (ii) $1/2 < \theta < 1$ and i is a large donor, we can solve the above matrix with Cramer's Rule and get:

$$\frac{\partial \hat{c}_i^n}{\partial o^n} = \left(- \underset{(+)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n}} \underset{(-)}{\frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2}} + \underset{(+)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}} \underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial o^n}} \right) \Bigg/ \left(\underset{(-)}{\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2}} \underset{(-)}{\frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2}} - \underset{(+)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}} \underset{(-)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right).$$

We can conclude that $\frac{\partial \hat{c}_i^n}{\partial o^n} > 0$.

However, we cannot pin down the sign of:

$$\frac{\partial \hat{c}_j^n}{\partial o^n} = \left(- \underset{(-)}{\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2}} \underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial o^n}} + \underset{(-)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n}} \underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right) \Bigg/ \left(\underset{(-)}{\frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2}} \underset{(-)}{\frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2}} - \underset{(-)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}} \underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right).$$

That is, when (iii) $0 < \theta < 1/2$ and j is a large donor or (iv) $1/2 < \theta < 1$ and j is a small donor, the effect on donor j is ambiguous.

■

The partial effect of an increase in the project quality parameter o^n is to raise every donor's contribution. This is because the marginal utility of own contribution is increased by the quality parameter. Since marginal utility is decreasing, a donor needs to increase his contribution as the value of the project increases to equate the marginal utility of own contribution with the marginal cost of contribution. We call this the *satiation effect.* There are offsetting strategic interactions involved, however, that complicate the the comparative statics.

In the case where donors are motivated by Personal Benefits, the small donor sees the large donor's contribution as a strategic complement. Therefore, the total effect is that the small donor will increase his contribution. For the large donor, however, the strategic substitute effect (with an increased contribution from the small donor) counters the satiation effect and the total effect is ambiguous.

In the case where donors are motivated by Reputation, the large donor sees the small donor's contribution as a strategic complement. Therefore, the total effect is that the large donor will increase contributions. For the small donor, however, the strategic substitute effect counters the satiation effect and the total effect is ambiguous.

Theorem 4. *(Impact of a_i^n)*[7]

(i) In the Personal Benefit case, an increase in the small donor's ability will increase the contribution of the small donor and decrease the contribution of the large donor.

(ii) In the Personal Benefit case, an increase in the large donor's ability will increase the contributions of both donors.

(iii) In the Reputation case, an increase in the small donor's ability will increase the contributions of both donors.

(iv) In the Reputation case, an increase in the large donor's ability will increase the contribution of the large donor and decrease the contribution of the small donor.

Proof/

Differentiate the first order conditions

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0,$$
$$\frac{\partial v_j^n}{\partial c_j^n} - \frac{g_j'}{a_j^n} = 0,$$

with respect to a_i^n and we have

$$\begin{pmatrix} \frac{\partial^2 v_i^n}{\left(\partial c_i^n\right)^2} & \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n} \\ \frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n} & \frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2} \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{c}_i^n}{\partial a_i^n} \\ \frac{\partial \hat{c}_j^n}{\partial a_i^n} \end{pmatrix} = \begin{pmatrix} -\frac{g_i'}{\left(a_i^n\right)^2} \\ 0 \end{pmatrix}.$$

[7] Note that although when a_i^n increases the contribution c_i^n certainly increases, it is not clear if effort ($e_i^n = c_i^n / a_i^n$) does as well. It may be that agents take advantage of their higher abilities by economizing on effort while still increasing their effective contribution.

When (i) $0 < \theta < 1/2$ and i is a small donor or (iv) $1/2 < \theta < 1$ and i is a large donor, Cramer's Rule solves to:

$$\frac{\partial \hat{c}_i^n}{\partial a_i^n} = -\underset{(+)}{\frac{g_i'}{(a_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} \Big/ \left(\underset{(-)}{\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} - \underset{(+)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right)$$

$$\frac{\partial \hat{c}_j^n}{\partial a_i^n} = \underset{(+)}{\frac{g_i'}{(a_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \Big/ \left(\underset{(-)}{\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} - \underset{(+)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right).$$

We can conclude that $\frac{\partial \hat{c}_i^n}{\partial a_i^n} > 0$ and $\frac{\partial \hat{c}_j^n}{\partial a_i^n} < 0$.

When (ii) $0 < \theta < 1/2$ and i is a large donor or (iii) $1/2 < \theta < 1$ and i is a small donor, Cramer's Rule solves to:

$$\frac{\partial \hat{c}_i^n}{\partial a_i^n} = \underset{(+)}{\frac{g_i'}{(a_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} \Big/ \left(\underset{(-)}{\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} - \underset{(-)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}}\;\underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right)$$

$$\frac{\partial \hat{c}_j^n}{\partial a_i^n} = \left(\underset{(+)}{\frac{g_i'}{(a_i^n)^2}}\;\underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \Big/ \underset{(-)}{\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}}\;\underset{(-)}{\frac{\partial^2 v_j^n}{(\partial c_j^n)^2}} - \underset{(-)}{\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}}\;\underset{(+)}{\frac{\partial^2 v_j^n}{\partial c_j^n \partial c_i^n}} \right).$$

We can conclude that $\frac{\partial \hat{c}_i^n}{\partial a_i^n} > 0$ and $\frac{\partial \hat{c}_j^n}{\partial a_i^n} > 0$.

■

The intuitive interpretation is as follows:

(i) In the Personal Benefit case, an increase in the small donor's ability will increase the contribution of the small donor and decrease the contribution of the large donor. The small donor finds it cheaper to contribute and so will contribute more. This crowds out the large donor's contribution (the effect of strategic substitutes).

(ii) In the Personal Benefit case, an increase in the large donor's ability will increase the contributions of both donors. The large donor finds it cheaper to contribute and so contributes more. Since this is a strategic complement to the small donor's contribution, he also increases his contributions.

(iii) In the Reputation case, an increase in the small donor's ability will increase the contributions of both donors. The small donor finds it cheaper to contribute on the margin and so contributes more and gets a larger share of the responsibility. Since this would leave contributions more even, the larger donor finds that it is cheaper to increase his share of responsibility as well. Thus, he responds by increasing his contribution (the effect of strategic complements).

(iv) In the Reputation case, an increase in the large donor's ability will increase the contribution of the large donor and decrease the contribution of the small donor. The large donor finds it cheaper to increase his share of responsibility and so he increases his contribution. Since this would leave contributions more uneven, the smaller donor finds that it is more expensive to increase his share of responsibility. Thus, he responds by decreasing his contribution (the effect of strategic substitutes).

5.2 Symmetric Games

In this subsection, we examine a symmetric game with $a_i^n = \bar{a}^n$ and $g_i' = \bar{g}'$ for all $i \in I$ and $n > 2$. There is a symmetric equilibrium to this game where $c_i^n = \bar{c}^n$ for all $i \in I$. Plugging the symmetric values into the first order condition

$$\frac{\partial v_i^n}{\partial c_i^n} - \frac{g_i'}{a_i^n} = 0,$$

we can solve $\bar{c}^n$ as follows.

$$(o^n)^{1-\theta} (\bar{c}^n)^{\theta-1} (I\bar{c}^n)^{-2\theta} (\theta (I-1) \bar{c}^n + (1-\theta) \bar{c}^n) = \frac{\bar{g}'}{\bar{a}^n},$$

$$(\bar{c}^n)^\theta = \frac{\bar{a}^n (o^n)^{1-\theta} I^{-2\theta} (\theta (I-1) + (1-\theta))}{\bar{g}'}.$$

We will examining comparative statics starting from this symmetric equilibrium. In this case, everyone is a small donor. Symmetry implies that for all i, $\frac{\partial^2 v_i^n}{(\partial c_i^n)^2} \equiv SAT < 0$ has the same value. SAT represents the size of the satiation effect as higher absolute values imply that marginal utility decreases more quickly. In the same way, for all $i \neq j$, $\frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n} \equiv STR \in \Re$ has the same value. STR represents the *strategic effect* since positive values imply that the marginal utility of an agent's contributions goes up when other agents contribute more (strategic complements), while negative values imply that the marginal utility of an agent's contributions goes down when other agents increase their contributions (strategic substitutes).

Theorem 5. *(Impact of o^n)*

(i) In the Personal Benefit case, if the impact parameter goes up, all players will decrease contributions if $SAT + (I-1)\,STR > 0$ and will increase contributions if $SAT + (I-1)\,STR < 0$,

(ii) In the Reputation case, if the impact parameter goes up, all players will increase contributions.

Proof/

At a symmetric equilibrium, the derivative $\frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n} \equiv \gamma > 0$ takes the same value for all i, and $\frac{\partial \hat{c}_i^n}{\partial o^n} \equiv \eta \in \Re$ also takes the same value for all i. The system of equations defining the impact of o^n in Section 5,

$$\frac{\partial^2 v_i^n}{(\partial c_i^n)^2}\frac{\partial \hat{c}_i^n}{\partial o^n} + \sum_{j \neq i} \frac{\partial^2 v_i^n}{\partial c_i^n \partial c_j^n}\frac{\partial \hat{c}_j^n}{\partial o^n} + \frac{\partial^2 v_i^n}{\partial c_i^n \partial o^n} = 0 \text{ for all } i,$$

reduce to the following equation:

$$(SAT + (I-1)\,STR)\,\eta + \gamma = 0,$$

or

$$\eta = \frac{-\gamma}{SAT + (I-1)\,STR}.$$

(i) When $0 < \theta < 1/2$, we have $SAT < 0$ and $STR > 0$. If $SAT + (I-1)\,STR > 0$ we have $\eta < 0$. If $SAT + (I-1)\,STR < 0$, we have $\eta > 0$.

(ii) When $1/2 < \theta < 1$, we have $SAT < 0$ and $STR < 0$. It is therefore clear that $\eta > 0$.

■

An increase in the project quality increases the marginal utility of contribution. A donor needs to increase his contribution to equate marginal utility of own contribution with the marginal cost of contribution, since marginal utility is decreasing (the satiation effect). Facing a change in equilibrium contributions, the strategic effects are coming from all $I-1$ donors and play a more explicit role here than in the two-person game.

In the Personal Benefit case, donors see each others' contributions as strategic complements ($STR > 0$). The satiation effect and the strategic effect go the opposite directions. When the strategic effect is larger ($SAT + (I-1)\,STR > 0$), a donor needs to decrease his contribution to equilibrate marginal utility of contribution and marginal cost of contribution, compensating for the increase in the marginal utility of contribution. When the satiation effect is larger ($SAT + (I-1)\,STR < 0$), a donor needs to increase his contribution to equilibrate marginal utility of contribution and marginal cost of contribution, compensating for the increase in the marginal utility of contribution.

In the Reputation case, donors see each others' contributions as strategic substitutes ($STR < 0$). The satiation effect and the strategic effect work in the same direction. For increased marginal utility of contribution, a donor needs to increase his contribution to equilibrate, compensating for the increase in the marginal utility of contribution.

Theorem 6. *(Impact of a_i^n)*

(i) In the Personal Benefit case, if $SAT + (I-1)\,STR < 0$, all players will increase contributions in response to an increase in the ability of one player.

(ii) In the Personal Benefit case, if $SAT + (I-1)\,STR > 0$, a player will decrease contribution if $SAT + (I-1)\,STR < -\frac{SAT-STR}{I-1}$ and will increase contribution if $SAT + (I-1)\,STR > -\frac{SAT-STR}{I-1}$ in response to an increase in own ability, while all other players will decrease contributions.

(iii) In the Reputation case, if $SAT - STR < 0$, a player will increase his contribution in response to an increase in his own ability, while other players will decrease contributions.

(iv) In the Reputation case, if $SAT - STR > 0$, a player will decrease his contribution in response to an increase in his own ability, while all other players will increase contributions.

Proof/

Suppose a_1^n increases. At equilibrium, for all $j \neq 1$, $\frac{\partial \hat{c}_j^n}{\partial a_1^n} = \mu$ takes the same value. The system of equations:

$$\frac{\partial^2 v_1^n}{(\partial c_1^n)^2} \frac{\partial \hat{c}_1^n}{\partial a_1^n} + \sum_{j \neq 1} \frac{\partial^2 v_1^n}{\partial c_1^n \partial c_j^n} \frac{\partial \hat{c}_j^n}{\partial a_1^n} = -\frac{g_1'}{(a_1^n)^2},$$

$$\frac{\partial^2 v_j^n}{\left(\partial c_j^n\right)^2} \frac{\partial \hat{c}_j^n}{\partial a_1^n} + \sum_{k \neq j} \frac{\partial^2 v_j^n}{\partial c_j^n \partial c_k^n} \frac{\partial \hat{c}_k^n}{\partial a_1^n} = 0 \text{ for all } j \neq 1$$

becomes:

$$SAT \frac{\partial \hat{c}_1^n}{\partial a_1^n} + (I-1)\, STR \mu = -\frac{g'}{(a_1^n)^2}, \tag{foc1}$$

$$(SAT + (I-2)\, STR)\, \mu + STR \frac{\partial \hat{c}_1^n}{\partial a_1^n} = 0. \tag{foc2}$$

Subtract Equation (foc2) from Equation (foc1), we have

$$(SAT - STR) \left(\frac{\partial \hat{c}_1^n}{\partial a_1^n} - \mu \right) = -\frac{g'}{(a_1^n)^2}. \tag{*}$$

Multiply Equation (foc2) by $(I-1)$ and add Equation (foc1) to get:

$$(SAT + (I-1)\, STR) \left(\frac{\partial \hat{c}_1^n}{\partial a_1^n} + (I-1)\, \mu \right) = -\frac{g'}{(a_1^n)^2}. \tag{**}$$

The above (*) and (**) solve to:

$$\mu = -\frac{g'}{I\,(a_1^n)^2} \left(\frac{1}{SAT + (I-1)\, STR} - \frac{1}{SAT - STR} \right).$$

$$\frac{\partial \hat{c}_1^n}{\partial a_1^n} = -\frac{g'}{I\left(a_1^n\right)^2}\left(\frac{1}{SAT+(I-1)\,STR}+\frac{I-1}{SAT-STR}\right).$$

(1) When $0 < \theta < 1/2$, we have $SAT < 0$ and $STR > 0$. Thus, $SAT + (I-1)\,STR > SAT - STR$. There are two further cases: (i) If $SAT+(I-1)\,STR < 0$, we have $\frac{1}{SAT+(I-1)STR} - \frac{1}{SAT-STR} < 0$ and $\mu > 0$, and $\frac{1}{SAT+(I-1)STR} + \frac{I-1}{SAT-STR} < 0$ and $\frac{\partial \hat{c}_1^n}{\partial a_1^n} > 0$. (ii) If $SAT + (I-1)\,STR > 0$, we have $\frac{1}{SAT+(I-1)STR} - \frac{1}{SAT-STR} > 0$ and $\mu < 0$. When $SAT + (I-1)\,STR < -\frac{SAT-STR}{I-1}$, $\frac{\partial \hat{c}_1^n}{\partial a_1^n} < 0$; when $SAT + (I-1)\,STR > -\frac{SAT-STR}{I-1}$, $\frac{\partial \hat{c}_1^n}{\partial a_1^n} > 0$.

(2) When $1/2 < \theta < 1$, we have $SAT < 0$ and $STR < 0$. From (foc2), we have that $\frac{\partial \hat{c}_1^n}{\partial a_1^n}$ and μ have opposite signs. Since $SAT + (I-1)\,STR < SAT - STR$, the sign of $\frac{1}{SAT+(I-1)STR} - \frac{1}{SAT-STR}$ depends on the sign of $SAT - STR$. (i) If $SAT - STR < 0$, we have $\mu < 0$, and $\frac{\partial \hat{c}_1^n}{\partial a_1^n} > 0$. (ii) If $SAT - STR > 0$, we have $\mu > 0$, and $\frac{\partial \hat{c}_1^n}{\partial a_1^n} < 0$.

■

The interactions of the satiation effect and the strategic effect are complicated in the face of a change in ability. An increase in donor 1's ability parameter reduces his marginal cost of contribution. The derivatives of the first order conditions say that donor 1 is evaluating own satiation effect and the strategic effect from all other donors to compensate for his decreased marginal cost of contribution, while other donors are equating own satiation effect and the strategic effect coming from all other donors.

In the Personal Benefit case, donors see each others' contributions as strategic complements ($STR > 0$). (1) When the satiation effect is larger than the strategic effect ($SAT + (I-1)\,STR < 0$), donor 1 will increase his contribution to equilibrate marginal utility of contribution and marginal cost of contribution, compensating for his decreased marginal cost of contribution. Facing an increase in donor 1's contribution, all other donors will increase contributions. (2) When the strategic effect is larger than the satiation effect ($SAT + (I-1)\,STR > 0$) but not too large ($SAT + (I-1)\,STR < -\frac{SAT-STR}{I-1}$), donor 1 will increase his contribution to equilibrate. When the strategic effect is very large ($SAT + (I-1)\,STR > -\frac{SAT-STR}{I-1}$), donor 1 will decrease contribution. Other donors, however, also will decrease contributions.

In the Reputation case, the comparative statics are determined by the size of the satiation effect and the size of the strategic effect from one donor ($STR < 0$). When the satiation effect is larger than the one-person strategic effect ($SAT - STR < 0$), donor 1 increases contribution, and all other donors decrease contributions because donor 1's contribution is a strategic substitute. When the one-person strategic effect is larger ($SAT - STR > 0$), donor 1 decreases his contribution, and all other players increase contributions because donor 1's contribution is a strategic substitute.

6. Summary of Results

Small Donor/Personal Benefit:

1. Strategic Complements
2. (Two-person case) $o^n \uparrow \quad \rightarrow \quad c_i \uparrow$.
3. (Two-person case) $a_i^n \uparrow \quad \rightarrow \quad c_i \uparrow, c_j \downarrow$
4. (Symmetric case) $o^n \uparrow \quad \rightarrow \quad c_i ?$
5. (Symmetric case) $a_i^n \uparrow \quad \rightarrow \quad c_i \uparrow, c_j \uparrow$

Small Donor/Reputation:

1. Strategic Substitutes
2. (Two-person case) $o^n \uparrow \quad \rightarrow \quad c_i ?$
3. (Two-person case) $a_i^n \uparrow \quad \rightarrow \quad c_i \uparrow, c_j \uparrow$
4. (Symmetric case) $o^n \uparrow \quad \rightarrow \quad c_i \downarrow$
5. (Symmetric case) $a_i^n \uparrow \quad \rightarrow \quad c_i ?, c_j ?$

Large Donor/Personal Benefit:

1. Strategic Substitutes
2. (Two-person case) $o^n \uparrow \quad \rightarrow \quad c_i ?$
3. (Two-person case) $a_i^n \uparrow \quad \rightarrow \quad c_i \uparrow, c_j \uparrow$

Large Donor/Reputation:

1. Strategic Complements
2. (Two-person case) $o^n \uparrow \quad \rightarrow \quad c_i \uparrow$
3. (Two-person case) $a_i^n \uparrow \quad \rightarrow \quad c_i \uparrow, c_j \downarrow$

7. Conclusion

Voluntary contributions to public projects are very significant at every level of society. Husbands and wives contribute to raising children and keeping a household, authors, referees and editors contribute to academic journals, believers contribute to churches, and software engineers contribute to FOSS projects, just to mention a few examples. Many explanations have been proposed in the economics and other literatures. In this paper, we have taken an integrative approach and considered the three most often discussed motivations for voluntary contributions in the context of an economy with many agents having heterogeneous abilities who must choose how to allocate their resources over private consumption and a variety of different public projects.

We show that Nash equilibrium exists in general. We distinguish between donors who are small and large in equilibrium. We find that contributions are strategic complements for small donors motivated mainly by Personal Benefits, and large donors motivated by Reputation, and are strategic substitutes for small donors motivated mainly by Reputation, and large donors motivated by Personal Benefits. We also carry out comparative static analysis of a two person economy and a large symmetric economy in which all agents have equal abilities. The results are summarized in the matrix given in the previous section. While there is no ambiguity from a mathematical standpoint, many of these findings were contrary to our initial expectations. This shows how complicated the interactions of the Reputation, Warm Glow and Personal Consumptions motivations to contribute can be even in a relatively simple model.

There are three areas at least that merit deeper study. The first is modeling more explicitly the details of how and why contributors benefit from reputation. Signaling one's quality is certainly one of the reasons reputation is valuable, but it would be interesting to see this in the context of a tournament model instead of one in which rewards are proportional to reputation. Games of status and gifting economies are also understudied in the context of voluntary contributions. Second, while there is a small literature on the modularity of FOSS projects (that is, the fact that they are made up of different components that are differently valued by different users), this is still an under appreciated feature of FOSS and other voluntary projects in the economics literature. Finally, something we completely ignore is the question of leadership. Why someone chooses to lead a FOSS project, what constraints leaders operate under, and what makes a leader of any voluntary enterprise successful are questions of great empirical importance. Without leaders, these projects would not exist and so understanding how entry and exit take place in this context is a subject that greatly merits additional study.

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