Endogenous Enfranchisement when Groups' Preferences Conflict†

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and

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Abstract

In their seminal paper, Aumann, Kurz and Neyman (1987) found the surprising result that the choice of public goods levels in a democracy is not affected by the distribution of voting rights. This implies that groups of individuals should not value the franchise. This conclusion, however, does not correspond to what we commonly observe. We propose a new model to address the question of enfranchisement. The main feature of our model is that it takes into account natural affinities, such as religion or class, which may exist between voters. This allows us to show that while individuals may not value the vote, they nonetheless value the franchise. We also show that in the presence of nonconvexities, it is more likely that the group in power will grant the franchise when preferences are severely opposed.

Keywords: Franchise, enfranchisement, Shapley value, voting.

1. Introduction

In their seminal paper, Aumann, Kurz and Neyman (1987) found the surprising result that the choice of public goods levels in a democracy is not affected by the distribution of voting rights. This implies that groups of individuals should not value the franchise. This conclusion, however, does not seem to be supported by our everyday observation. Protracted and costly struggles to gain the franchise are one of the hallmarks of modern history. Consider, for example, the recent situation in South Africa or the struggles of blacks and women in the United States earlier in this century.

An important feature of Aumann, *et al.* is that voters are treated as individuals who have no particular relationship to one another. We take an alternative approach in this paper. We assume that there exist exogenously given groups of voters whose preferences and interests are correlated. For example, there may be natural affinities between voters of the same gender, race, religion, class or age. This explicit treatment of interest groups allows us to address questions surrounding the value of the vote from a new perspective. In particular, we show that individuals may value the right to vote independently of how much they value the act of voting itself. We also explore conditions under which an empowered group should extend the right to vote to other groups.

The literature addressing the question of how people should value the vote is extensive. One of the most important early works in this area (Downs 1957) suggested that since the probability of any one voter affecting the outcome of an election is negligible in a large democracy, rational individuals should not vote at all. This suggests that they should not value the vote at all either. More sophisticated game theoretic models in the same spirit conclude that voters should in fact vote in positive numbers, but these results seem to be very sensitive to the information structure of the underlying game. See Ledyard (1981, 1984) and Palfrey and Rosenthal (1983, 1985). It is not completely clear how much voters should value the franchise in these cases, but the suggestion is not very much.

Another strand of the literature tries to motivate voting by postulating that voters derive benefits from the act of voting itself independently of the outcome of the vote. In other words, people vote because they have a sense of civic duty. The main criticisms of this approach are that it removes politics from political action and fails to explain observed behavior. For excellent surveys, see Mueller (1989) and Uhlaner (1993). Note that in these cases, the value of the vote is exogenously assumed rather than endogenously derived. It may be possible, however, to provide an evolutionary foundation for why public-spirited preferences like this come into being (Conley and Toossi 1999).

Another approach that is closer in spirit to the current paper embeds individual decision making in a social structure. Uhlaner (1989a, 1989b) develops a model where individuals belong to groups. Members of a group identify with each other and have similar preferences. Group leaders then play a crucial role in coordinating the actions of individuals within groups. Thus, leaders play the role of intermediaries between citizens and candidates. The leaders' behavior depends on instrumental calculations by virtue of the influence of a group rather than a negligible individual on the outcome of an election. This cleverly reinstates the relationship between the decision to vote and politics.

All of these approaches are different from that of Aumann *et al.* (1977, 1983, and 1987). In this line of research, the authors abstract from the issue of why people vote and simply assume that people will vote if they are given the right to do so. They then develop a general equilibrium model to analyze the effect of the distribution of voting rights on the choice of public goods in an economy. The choice of public goods is then determined by a simple majority of citizens. One might think that the preferences of the noncitizens should account for nothing in the choice of public goods. However, this is not what Aumann *et al.* (1987) found. The main result they derive is that the choice of public goods depends exclusively on the fundamentals of the economy, namely the distribution of preferences and endowments of the entire population. In particular, the choice of public goods does not depend on who has the right to vote.

The explanation for this surprising result seems to lie in their choice of using the

Shapley value to analyze the effect of different distributions of voting rights on the allocation of public goods in an economy. The Shapley value takes the worth of voters as depending on both their chance of affecting the outcome and as generators of utility. In a public goods economy, even if a voter transforms a minority into a majority, the total utility of the coalition he joins barely changes since he was enjoying the public goods even when he was in the minority group. Hence, the marginal contribution of any voter is negligible in a public goods framework. In a private goods economy on the other hand, the majority can exclude the minority from consuming the good they voted for. Hence, when a marginal voter transforms a minority into a majority, his contribution to the aggregate utility is significant and not just marginal as in the public goods case. Therefore the Shapley value depends on the distribution of voting rights in a private goods framework whereas it is independent of the distribution of voting rights in a public goods framework. This is why Aumann and Kurz (1977) found that the vote is of central importance in redistributional questions while Aumann *et al.* (1987) showed that the vote has little relevance in the choice of public goods.

A tangentially related literature in political science takes up the issue of the effect of enfranchisement of groups of voters on the growth of government. Kenny (1978), Husted and Kenny (1997), and Meltzer and Richard (1981) analyze the effect of the expansion of the voting franchise on the level of government spending. The conclusions turn out to depend on the relationship between the elasticity of substitution between government services and private goods and the income elasticity for government services.

A crucial property that drives Aumann *et al.* result is the property of random coalitions in the formula of the Shapley value. Indeed, every random coalition drawn from a continuum of heterogeneous agents will be statistically representative with probability one. In particular, this implies that a coalition of 49% of voters and a coalition of 51% of voters roughly enjoy the same level of utility in a public goods economy since they will vote for the same allocation of public goods. However, the assumption of random coalitions is very restrictive. In particular, using the Shapley value explicitly rejects the notion that voters with affinities will form a coalition. The possibility that exogenously given affinities exist among voters and these might affect the probability that various coalitions form has important implications.

In this paper, we explicitly take into account these affinities.[1] More specifically, we explore a model with two groups of individuals, the group in power and the unenfranchised group. Individuals within each group have correlated interests. There is an exogenous random generation of policy proposals that produce nonexcludable benefits to individuals from each group if passed. Preferences between the two groups over these proposals may be positively or negatively correlated. Within this framework, we analyze the following question: Is it rational for the unenfranchised group to engage in costly threats in order to gain the right to vote? The answer depends in part on the degree to which the interests of the two groups clash.

A second question we explore is why the currently enfranchised group would extend this privilege to other groups. It is especially puzzling that the franchise is expanded even when interests conflict. To examine this question, we extend our model to allow the unenfranchised group to impose costs on the group in power if the franchise is not granted. This might take the form of civil unrest or civil disobedience. To make their threats credible, the unenfranchised group must precommit itself by organizing and purchasing what is needed to carry out these actions before the group in power makes its decision. Once everything is in place, it is costless to actually carry out the threats. The group in power then faces a tradeoff in its decision to grant the franchise. On one hand, granting the right to vote affects them in a negative way since some proposals that they favor may no longer pass. On the other hand, by not granting the franchise, they may incur substantial losses resulting from riots and other forms of protest. The unenfranchised group must also decide whether to engage in costly threats by comparing the costs of organizing these protests and the benefits from having the franchise. We model these strategic interactions as a game of complete information. The resulting game is analyzed in terms of subgame perfect equilibrium. We then

[1] This paper is a modification of one the chapters of the Ph.D. thesis of the second author (Temimi 1996) and extends an idea contained in Wooders (1994).

derive conditions under which the resulting equilibrium involves the franchise being granted.

The most important result we derive is that while individuals may not value the vote, they nonetheless value the franchise. It is a general result of our analysis that the stronger the unenfranchised group, the higher the probability of having the franchise as an equilibrium outcome. The relationship between the likelihood of the franchise being granted in equilibrium and the degree of preferences conflict between the two groups depends on the characteristics of the threat technology. More specifically, if the threat technology is characterized by constant returns to scale, then we cannot say whether it is more likely or less likely that the franchise will be granted as the degree of preferences conflict increases. However, if economies of scale in the threat technology are present, then the probability of having the franchise as an equilibrium outcome increases with the degree of preferences' conflict. This is especially interesting since there are typically large set-up costs in organizing groups' activities before the group can be effective.

The rest of the paper is organized as follows. In section 2, we present a formal description of the model. In section 3, we investigate the case where the franchise is not granted. In section 4 we analyze the case of franchise. In section 5, we discuss the equilibria of the game. In the final section we present some concluding remarks.

2. The Model

We consider an economy populated by two groups of individuals, A and B. Group A is the enfranchised group and must decide whether to enfranchise B. Each group has a continuum of agents uniformly distributed on $(0, \theta_A)$ and $(0, 1-\theta_A)$ respectively. The measure of the whole set of agents is normalized to unity, and each individual has measure zero. Let there be an exogenous random generation of policy proposals, which if passed, would generate nonexcludable benefits (or losses) to individual agents from both groups. For each policy proposal, these benefits are summarized by two statistically correlated benefit functions $B_{i_A}(t)$ and $B_{i_B}(t)$, for A and B respectively (see figure 1).

Figure 1 about here

$$B_{i_A}(t) = a(i_A - t) \; ; \qquad t \in (0, \theta_A). \tag{1}$$

$$B_{i_B}(t) = a(i_B - t) \; ; \qquad t \in (0, 1 - \theta_A). \tag{2}$$

where i_A and i_B are the horizontal intercepts of $B_{i_A}(t)$ and $B_{i_B}(t)$, a is their common slope, and t is an index for individuals. The specific form of the correlation will be specified later. The benefit functions $B_{i_A}(t)$ and $B_{i_B}(t)$ are assumed to be linear for analytical simplicity. Once the slope a is fixed, the horizontal intercepts i_A and i_B completely identify $B_{i_A}(t)$ and $B_{i_B}(t)$. Therefore, we will denote proposals by their corresponding benefit functions' horizontal intercepts. For each proposal, we rank individuals from each group as follows: the individual with the highest benefit is to the extreme left, while the one to the extreme right derives the lowest benefit. Therefore the benefit functions $B_{i_A}(t)$ and $B_{i_B}(t)$ are nonincreasing in t.

Suppose that i_A is drawn from a uniformly distributed random variable I_A.

$$I_A \sim U(0, \theta_A). \tag{3}$$

Note that if $i_A = 0$, every individual from group A derives negative benefit from the policy proposal. If $i_A = \frac{\theta_A}{2}$, one half of group A derive positive benefits. Finally, if $i_A = \theta_A$, every member from A gets positive utility. The political procedure we consider is simple majority rule. This means that a proposal passes if it is approved by at least one half of the voters. For instance, if only group A is allowed to vote, a proposal will pass if and only if $i_A \geq \frac{\theta_A}{2}$. If both groups vote, a proposal will pass if and only if $i_A + i_B \geq \frac{1}{2}$.

Figure 2 about here

We consider conflicting preferences over policy proposals between the two groups. For this purpose, we first generate a random variable $I_{\overline{A}}$ negatively correlated with I_A in the following sense (see figure 2). If every member from group A derives negative benefit from a certain proposal ($i_A = 0$), then everyone from B derives positive benefit from the same proposal ($i_{\overline{A}} = 1 - \theta_A$). Similarly, if a proposal is favored by all members of A ($i_A = \theta_A$), it is opposed by every member of B ($i_{\overline{A}} = 0$). For this purpose, we define $I_{\overline{A}}$ as follows:

$$I_{\overline{A}} = \frac{\theta_A - 1}{\theta_A} I_A + 1 - \theta_A. \tag{4}$$

It follows that $I_{\overline{A}}$ is uniformly distributed on $(0, 1 - \theta_A)$.

$$I_{\overline{A}} \sim \ U(0, 1 - \theta_A). \tag{5}$$

The benefits of group B will be a convex combination of this strictly negatively correlated variable and an uncorrelated random variable I_C. Hence, we define I_B as follows:

$$I_B = \alpha I_{\overline{A}} + (1 - \alpha) I_C, \tag{6}$$

where

$$I_C \ \sim U(0, 1 - \theta_A), \tag{7}$$

and $\alpha \in (0, 1)$. As α increases to one, I_A and I_B become strongly negatively correlated in the sense described above. When α goes to zero the two intercepts are uncorrelated and thus preferences of the two groups over proposals are independent. Hence, by varying α, we control the degree of correlation of preferences between the two groups. Note that this transformation guarantees that I_B lies in $(0, 1 - \theta_A)$. Substituting for $I_{\overline{A}}$ in (6), leads to the following:

$$I_B = \alpha \frac{\theta_A - 1}{\theta_A} I_A + \alpha(1 - \theta_A) + (1 - \alpha) I_C. \tag{8}$$

The total surplus accruing to group A from a proposal i_A, $TB(i_A)$, is obtained by integrating the benefit function $B_{i_A}(t)$ over the set of individuals of group A. Hence, it follows that:

$$\begin{aligned} TB(i_A) &= \int_0^{\theta_A} B_{i_A}(t)dt \\ &= \int_0^{\theta_A} a(i_A - t)dt \\ &= \frac{a\theta_A(2i_A - \theta_A)}{2} \end{aligned} \tag{9}$$

Note that $TB(i_A) \geq 0$ if and only if $i_A \geq \frac{\theta_A}{2}$.

Similarly, the total surplus to group B from a proposal i_B, $TB(i_B)$, is obtained by integrating $B_{i_B}(t)$ over the set of individuals of group B. Hence, we have:

$$\begin{aligned} TB(i_B) &= \int_0^{1-\theta_A} B_{i_B}(t)dt \\ &= \int_0^{1-\theta_A} a(i_B - t)dt \\ &= \frac{a(\theta_A - 1)(1 - \theta_A - 2i_B)}{2} \end{aligned} \tag{10}$$

Note that $TB(i_B) \geq 0$ if and only if $i_B \geq \frac{1-\theta_A}{2}$.

In deciding whether to grant the franchise group A compares the *ex ante* expected total surplus from all proposals in the case of franchise and no franchise. We now proceed to compute the *ex ante* expected total surplus for each group and under different scenarios.

3. Social Surplus in the Case of No Franchise

In this section, we consider the scenario where group A does not grant the right to vote to group B (no franchise: abbreviated by NF). In this case, under simple majority rule, a proposal i_A passes if at least one half of group A approve it. Therefore, the *ex ante* expected total surplus for group A, $E_A TB^{NF}$, is obtained by integrating the total surplus $TB(i_A)$ weighted by the density function $f(i_A)$ over proposals for which more than one half of group A derive nonnegative benefits.

$$\begin{aligned} E_A TB^{NF} &= \int_{\frac{\theta_A}{2}}^{\theta_A} TB(i_A) f(i_A) di_A \\ &= \frac{a\theta_A^2}{8} \end{aligned} \tag{11}$$

where $f(i_A)$ is the p.d.f of I_A and $TB(i_A)$ is given by (9).

$$f(i_A) = \frac{1}{\theta_A}; \qquad i_A \in (0, \theta_A).$$

To determine the *ex ante* expected total surplus for group B, some careful analysis has to be undertaken to determine the range of integration. First, it follows from (7) and (8) that the conditional distribution of I_B given a proposal i_A is given by:

$$I_B/I_A = i_A \sim U\left(\alpha(1-\theta_A) + \alpha\frac{\theta_A - 1}{\theta_A} i_A, 1 - \theta_A + \alpha\frac{\theta_A - 1}{\theta_A} i_A\right) \tag{12}$$

Under no franchise, only the proposals $i_A \in (\frac{\theta_A}{2}, \theta_A)$ pass for group A since only members from A are allowed to vote. Therefore, the *ex ante* expected total surplus for group B under no franchise, $E_B TB^{NF}$, is obtained by integrating over all proposals i_B corresponding to each $i_A \in (\frac{\theta_A}{2}, \theta_A)$. For each i_A, the range of i_B is given by (12). Thus, we have

$$\begin{aligned} E_B TB^{NF} &= \int_{\frac{\theta_A}{2}}^{\theta_A} \int_{\alpha(1-theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B) f(i_A, i_B) di_B di_A \\ &= \frac{-a\alpha(\theta_A - 1)^2}{8} \end{aligned} \tag{13}$$

where $f(i_A, i_B)$ is the joint density function of (I_A, I_B) and $TB(i_B)$ is given by (10).

$$f(i_A, i_B) = \frac{1}{1-\alpha}\frac{1}{1-\theta_A}\frac{1}{\theta_A}$$

The joint density $f(i_A, i_B)$ is derived in an appendix. Note that (13) implies that

$$\frac{\partial E_B TB^{NF}}{\partial \alpha} \leq 0 \tag{14}$$

The above inequality says that as preferences become more diametrically opposed, the expected total surplus for group B decreases since proposals that are favored by B are less likely to pass and proposals that are disliked by B are more likely to pass.

4. Social Surplus in the Case of Franchise

The purpose of this section is to analyze the situation with franchise (abbreviated by F) whereby individuals from both groups have the right to vote. Under simple majority rule, a proposal passes if it is approved by at least one half of the total population. This is translated by the following inequality:

$$i_A + i_B \geq \frac{1}{2}. \tag{15}$$

But since given a proposal i_A, i_B is determined only up to a random shock, it follows that the passage of a proposal i_A will in general be probabilistic. This implicitly defines an approval function $A(i_A)$ that gives the probability that a proposal i_A will pass. It is roughly the proportion of i_B's belonging to $I_B(i_A)$ that satisfy $i_B \geq \frac{1}{2} - i_A$. We will use this approval function in computing the expected social surplus. The approval function depends on whether A is in the majority and on the degree of preferences' conflict α. The details are computationally involved and are relegated to an appendix. Here we present the basic results and comparative statics. The *exante* expected total surplus for group A in the case of franchise, $E_A TB^F$ is given by the following expression

$$E_A TB^F = \int_0^{\theta_A} TB(i_A)A(i_A)f(i_A)di_A \tag{16}$$

where $f(i_A)$ is the p.d.f of I_A, $A(.)$ is the approval function which is derived in an appendix and $TB(i_A)$ is given by (9). As mentioned above, there are several cases to consider. We will focus on the case where $\theta_A \leq \frac{1}{2}$ and $\alpha \leq \frac{1}{2(1-\theta_A)}$. The analysis for the other cases is identical and is relegated to an appendix. Taking the derivative of

(16) with respect to the parameter of preferences' conflict α, we find that

$$\frac{\partial E_A TB^F}{\partial \alpha} \leq 0 \tag{17}$$

The result in (17) simply says that as the degree of preferences' conflict α increases, the expected total surplus of group A decreases.

In a similar fashion, we compute, the *ex ante* expected total surplus , $E_B TB^F$, for group B. The computations are relegated to an appendix. Taking the derivative of $E_B TB^F$ with respect to α, we get the following

$$\frac{\partial E_B TB^F}{\partial \alpha} \leq 0 \tag{18}$$

It follows from (17) and (18) that as preferences become more conflicting, the expected total surplus of each group under franchise decreases.

5. Equilibrium Analysis

We now examine the question of why the currently enfranchised group would extend the franchise to the other group. It is especially puzzling that the franchise is expanded even when interests conflict. To examine this question, we will allow the possibility for the unenfranchised group to impose a cost on the group in power if the franchise is not granted. To make its threat credible, the unenfranchised group must precommit itself by organizing and purchasing what is needed to carry out the threats before the group in power makes its decision.

More specifically, we consider a game G with complete information. Let $G =< N, S, v >$ where $N = \{A, B\}$ is the set of players, S^i is the strategy set of player i and $v^i : S^A \times S^B = S \to R$ is the payoff function of player $i \in N$.

The sequence of moves is as follows: Group B moves first by choosing a threat level $T \in R^+$. If group B invests T dollars, they impose a loss of $C(T)$ dollars on group A if they carry out the threat. The function $C(T)$ summarizes the threat technology available to group B. Group A moves second, after observing $B's$ move, decides whether to grant the franchise. It is not uncommon to find situations that mimic this structure. Suppose for instance that the Israelis are negotiating in the Knesset whether to give the franchise to the Palestinians while the latter have already committed themselves to carry out the threats and assembled together waiting for the decision. If the franchise is not granted, they carry out the threats. However if they are given the right to vote they go home peacefully. A strategy for group A is a decision whether to grant the franchise for every possible threat level that may be chosen by group B. Formally, it is a mapping from R^+ to $\{F, NF\}$. Hence, the strategy space for A, S^A, is $S^A = \{f : R^+ \to \{F, NF\}\}$.

A strategy for group B is a choice of a threat level T. If they choose to invest T dollars, they inflict a cost of $C(T)$ dollars on group A if the threat is carried out. We will first assume that $C(T)$ is linear, $C(T) = K \cdot T$ where $K > 0$. We next describe the payoff functions of A and B.

If group B chooses a threat level T and group A does not grant the franchise (NF), then the payoffs of the two groups are given by (See Figure 3)

$$\big(v^A(T, NF), v^B(T, NF)\big) = \big(E_A T B^{NF} - C(T), E_B T B^{NF} - T\big)$$

Figure 3 about here

If group B chooses a threat level T and group A grants the franchise (F), then the payoffs are given by

$$\big(v^A(T, F), v^B(T, F)\big) = \big(E_A T B^F, E_B T B^F - T\big)$$

Note that once group B chooses a threat level $T > 0$, they incur the cost of preparation independently of whether they eventually carry out the threat. Group A suffers from the threats only if they decide not to grant the franchise. We now analyze the subgame perfect equilibria of G. If group B chooses a threat level T, group A will grant the franchise if the loss incurred from doing so is less than the loss they would suffer as a consequence of threats. This could be expressed as follows:

$$[E_A TB^{NF} - E_A TB^F][\alpha] \leq C(T) \tag{19}$$

The left-hand side of (19) is the expected loss for group A from granting the franchise, the right-hand side is the cost inflicted on group A if group B chooses a threat level T.

The unenfranchised group will engage in costly threats if the gain they would get from having the franchise is greater than the cost of engaging in threats. Formally, T has to satisfy the following inequality:

$$T \leq [E_B TB^F - E_B TB^{NF}][\alpha] \tag{20}$$

Combining conditions (19) and (20) leads to the following:

$$[E_A TB^{NF} - E_A TB^F][\alpha] \leq C(T) \leq C\left([E_B TB^F - E_B TB^{NF}][\alpha]\right) \tag{21}$$

The left-hand side of (21) is the loss to group A from granting the franchise. The right-hand side represents the maximum cost that group B is willing to impose on group A to get the franchise. Therefore, if there exists T satisfying (21), the subgame perfect equilibrium (henceforth SPE) outcome would be for B to choose the threat strategy (the equilibrium level of $C(T)$ would be $([E_A TB^{NF} - E_A TB^F][\alpha])$ and for A to grant the franchise. However, if the left-hand side of (21) is greater than the right-hand side, then the cost of threats that would induce A to grant the franchise is greater than the gain that B would get from having the franchise. Hence the equilibrium outcome would be for B to choose to be peaceful ($T = 0$) and for A not to grant the franchise.

We now analyze how the equilibrium evolves with the degree of preferences' conflict α. For this purpose, we analyze the behavior of (21) as the degree of preferences' conflict α varies. Taking the partial derivative with respect to α of the left-hand side of (21) leads to:

$$\frac{\partial [E_A TB^{NF} - E_A TB^F][\alpha]}{\partial \alpha} = -\frac{\partial [E_A TB^F][\alpha]}{\partial \alpha} \geq 0 \tag{22}$$

The result in (22) follows from (11) and (17) and says that the loss to group A from granting the franchise is increasing in α.

For the right hand side, using (14) and (18), after some derivations, we conclude that:

$$\frac{\partial C([E_B TB^F - E_B TB^{NF}][\alpha])}{\partial \alpha} = K\frac{\partial([E_B TB^F - E_B TB^{NF}][\alpha])}{\partial \alpha} \geq 0 \tag{23}$$

The result in (23) implies that as preferences become more conflicting, the expected total surplus of group B under franchise decreases at a lower rate than the expected total surplus under no franchise and hence the gain to group B from having the franchise is increasing in α which in turn makes group B willing to invest more in threats.

Figure 4 about here

It follows from (22) and (23) that the l.h.s and the r.h.s of (21) are increasing in α. However, they can cross several times (See Figure 4). Let α_n denote the n^{th} crossing of $C[E_B TB^F - E_B TB^{NF}][\alpha]$ and $[E_A TB^{NF} - E_A TB^F][\alpha]$ for $\alpha \leq 1/2(1-\theta_A)$. Let K^* denote the slope of the threat function which guarantees that $C([E_B TB^F - E_B TB^{NF}][\alpha])$ and $[E_A TB^{NF} - E_A TB^F][\alpha]$ cross at $\alpha = 1/2(1-\theta_A)$.

$$K^* = \frac{[E_A TB^{NF} - E_A TB^F][\frac{1}{2(1-\theta_A)}]}{[E_B TB^F - E_B TB^{NF}][\frac{1}{2(1-\theta_A)}]}. \tag{24}$$

Suppose that for any $\alpha < \frac{1}{2(1-\theta_A)}$,

$$K^*([E_B TB^F - E_B TB^{NF}][\alpha]) < [E_A TB^{NF} - E_A TB^F][\alpha] \tag{25}$$

If condition (25) is satisfied, then if $K = K^*$ the maximum cost that the unenfranchised group can impose on the group in power is too small to induce the group in power to grant the franchise for any α. Formally, it guarantees that there does not exist another crossing to the left of $\alpha = 1/2(1-\theta_A)$ when $K = K^*$. Let $C_0 = K\left([E_B TB^F - E_B TB^{NF}][0]\right)$ be the corresponding vertical intercept.

The following results extend from theorem 1 in the appendix.

Result 1.1 *If the unenfranchised group is sufficiently weak, then the franchise will not be granted in equilibrium independently of the degree of preferences conflict.*

Result 1.2 *If the unenfranchised group is sufficiently strong, then whether the franchise will be granted in equilibrium depends on the degree of preferences conflict α. However there is no monotonic relationship between the degree of preferences conflict and the franchise being the equilibrium outcome.*

The intuition behind Result 1.2 is as follows. As preferences become more conflicting, the unenfranchised group has more to gain from getting the right to vote. At the same time, the group in power has more to lose from giving the franchise. The equilibrium outcome depends on how the rate of change of the gain to group B and the rate of change of the loss to Group A from granting the franchise evolve as a function of α.

We now analyze the case where threats are rational at $\alpha = 0$. Suppose that for any $\alpha < \frac{1}{2(1-\theta_A)}$, the following condition is satisfied.

$$K^*([E_BTB^F - E_BTB^{NF}][\alpha]) > [E_ATB^{NF} - E_ATB^F][\alpha] \tag{26}$$

If condition (26) is satisfied, then the maximum cost that the unenfranchised group can impose on the group in power is bigger than the loss that group A suffers when they grant the franchise for any α when $K = K^*$. Formally (26) guarantees that the first crossing of $[E_ATB^{NF} - E_ATB^F][\alpha]$ and $C([E_ATB^F - E_BTB^{NF}][\alpha])$ occurs at $\alpha = \frac{1}{2(1-\theta_A)}$ when $K = K^*$.

The following result extends from Theorem 2 in the appendix.

Result 2.1 *If the unenfranchised group is very strong, then the franchise will be granted in equilibrium independently of the degree of preferences' conflict.*

So far, we considered linear threat technologies. We now investigate more general technologies. Assume that $C(T)$ has a general form and that it satisfies the following condition for any $\alpha \in (0, 1/2(1-\theta_A))$.

$$\frac{\partial C[E_BTB^F - E_BTB^{NF}][\alpha]}{\partial \alpha} > \frac{\partial [E_ATB^{NF} - E_ATB^F][\alpha]}{\partial \alpha} \tag{27}$$

Condition (27) is a sufficient condition for single crossing (if any) between the l.h.s and the r.h.s of (21) (See Figure 5). Let $C_0 = C([E_B TB^F - E_B TB^{NF}][0])$. Let C_0^* be the intercept that guarantees that $C[E_B TB^F - E_B TB^{NF}][\alpha]$ and $[E_A TB^{NF} - E_A TB^F][\alpha]$ are equal at $1/2(1-\theta_A)$.

Figure 5 about here

The following result extends from theorem 3 in the appendix.

Result 3.1 *Under sufficient increasing returns to scale in the threat technology, the likelihood of the franchise being granted in equilibrium is increasing in the degree of preferences' conflict α.*

Result 3.1 implies that when sufficient increasing returns in the threat technology are present, the probability of having the franchise as an equilibrium outcome increases with the degree of preferences conflict. As preferences become more conflicting, the unenfranchised group has more to gain from getting the franchise and hence is willing to invest more in threats. In the presence of increasing returns to scale, the more they invest in threats the more effective they become in imposing costs on the group in power and hence the likelihood that they will get the franchise increases. This is especially interesting since typically there are large set-up costs in organizing group's activities before the group can be effective. This implies that the empowered group may have an interest in mollifying the other group.

6. Concluding Remarks

This paper has examined the question of enfranchisement of groups of individuals. In our model, affinities between groups of people play a central role. Within this framework, we show that while individuals may not value the vote, they nonetheless value the franchise. We also show that if the threat technology is characterized by constant returns to scale, then it is unclear whether increased polarization makes enfranchisement less likely. However, we show that in the presence of large set-up costs in the threat activity, it is more likely for the franchise to be granted when the preferences are severely opposed. This implies that the empowered group may have an interest in placating the other group.

We have focused our attention on a static game with complete information. However, we observe that the group in power often grants the franchise after incurring considerable losses through time resulting from different forms of protests. The situation in South Africa is an example of this. In ongoing research, we extend this static model by introducing asymmetric information in a dynamic framework. When the two groups are uncertain about the strength of their opponents, equilibria involve dynamic selection. In other words, as time passes, each group becomes increasingly pessimistic about the other group's strength. Weak types drop early in the game while sufficiently strong types drop at infinity. We then introduce international support and sanctions for the group in power and the unenfranchised group respectively. We get equilibria indexed by the relative strength of the two parties. One possible extension would be to investigate the possibility of introducing international arbitration and how it would affect the underlying equilibria of our model.

Appendix

Derivation of the conditional distribution

To get $f(i_B/i_A)$, we use the distribution method.

$$\begin{aligned}
F(i_B/I_A = i_A) &= P\ (I_B \leq i_B \,/\, I_A = i_A) \\
&= P\left(\alpha\frac{\theta_A - 1}{\theta_A}i_A + \alpha(1-\theta_A) + (1-\alpha)I_C \leq i_B\right) \\
&= P\left(I_C \leq \frac{i_B - \alpha\frac{\theta_A - 1}{\theta_A}i_A - \alpha(1-\theta_A)}{1-\alpha}\right) \\
&= F_C\left(\frac{i_B - \alpha\frac{\theta_A - 1}{\theta_A}i_A - \alpha(1-\theta_A)}{1-\alpha}\right)
\end{aligned}$$

where F_C denotes the cumulative distribution of I_C. It then follows that

$$\begin{aligned}
f(i_B/i_A) &= \frac{\partial F(i_B/i_A)}{\partial i_B} \\
&= \frac{1}{1-\alpha}f_C\left(\frac{i_B - \alpha\frac{\theta_A - 1}{\theta_A}i_A - \alpha(1-\theta_A)}{1-\alpha}\right) \\
&= \frac{1}{1-\alpha}\,\frac{1}{1-\theta_A}
\end{aligned}$$

where f_C is the probability density function of I_C. Since I_C is uniformly distributed on $(\theta_A, 1)$, it follows that

$$f_C(i_C) = \frac{1}{1-\theta_A}$$

Derivation of the joint p.d.f:

We should now find the joint p.d.f of (I_A, I_B). Recall from the Bayesian formula that:

$$f(i_A, i_B) = f(i_B/i_A)f(i_A)$$

Hence,

$$f(i_A, i_B) = f(i_B/i_A)f(i_A) = \frac{1}{1-\theta_A}\frac{1}{1-\alpha}\frac{1}{\theta_A}$$

where

$$i_A \in (0, \theta_A)\,,\, i_B \in \left((1-\alpha)\theta_A + \alpha\frac{\theta_A - 1}{\theta_A}i_A + \alpha, 1 + \alpha\frac{\theta_A - 1}{\theta_A}i_A\right).$$

Derivation of the Approval Function

First, recall from (12) that the set of i_B's corresponding to a proposal i_A is given by

$$I_B(i_A) = \left(\alpha(1-\theta_A) + \alpha\frac{\theta_A - 1}{\theta_A} i_A, 1 - \theta_A + \alpha\frac{\theta_A - 1}{\theta_A} i_A\right)$$

Under simple majority rule, a proposal passes the vote if it is approved by at least one half of the total population. This is translated in the following condition:

$$i_A + i_B \geq \frac{1}{2}. \tag{15}$$

But since given a proposal i_A, i_B is determined only up to a random shock, it follows that the passage of a proposal i_A will in general be probabilistic. This implicitly defines an approval function $A(i_A)$ that gives the probability that a proposal i_A will pass. It is roughly the proportion of i_B's belonging to $I_B(i_A)$ that satisfy $i_B \geq \frac{1}{2} - i_A$. More specifically, there are several cases to consider:

1) $\theta_A \geq \frac{1}{2}$:

a) If

$$i_A < \frac{\theta_A - \frac{1}{2}}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}}$$

then $A(i_A) = 0$. For each proposal i_A in this range, there does not exist i_B's belonging to $I_B(i_A)$ such that $i_B \geq \frac{1}{2} - i_A$. Intuitively, proposals in this range are opposed by a sufficiently large number of group A that they will not pass even if they are strongly supported by group B.

b) If

$$\frac{\theta_A - \frac{1}{2}}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}} < i_A < \frac{\frac{1}{2} - \alpha(1-\theta_A)}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}}$$

then

$$A(i_A) = \int_{\frac{1}{2} - i_A}^{1 - \theta_A + \alpha\frac{\theta_A - 1}{\theta_A}} f(i_B/i_A) di_B$$

where $f(i_B/i_A)$ is given by

$$f(i_B/i_A) = \frac{1}{1-\alpha}\frac{1}{1-\theta_A}$$

c) If

$$\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}} < i_A < \theta_A$$

then

$$A(i_A) = \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} f(i_B/i_A)di_B = 1$$

The *exante* expected total surplus for group A in the case of franchise, E_ATB^F is given by the following expression

$$E_ATB^F = \int_0^{\theta_A} TB(i_A)A(i_A)f(i_A)di_A$$

where $f(i_A)$ is the p.d.f of I_A, $A(.)$ is the approval function and $TB(i_A)$ is given by (9). This is equivalent to

$$E_ATB^F = \int_{\frac{\theta_A-\frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_Bdi_A+$$

$$\int_{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\theta_A} \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_Bdi_A$$

where $f(i_A,i_B)$ is the joint p.d.f of (I_A,I_B) and $TB(i_A)$ is given by (9).

2) $\theta_A \leq \frac{1}{2}$

a) $\alpha \leq \frac{1}{2(1-\theta_A)}$

In this case, for any i_A

$$A(i_A) = \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}} f(i_B/i_A)di_B$$

and the ex ante expected total payoff is given by

$$E_ATB^F = \int_0^{\theta_A} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_Bdi_A$$

b) $\alpha \geq \frac{1}{2(1-\theta_A)}$

b1) If

$$0 \leq i_A \leq \frac{\frac{1}{2} - \alpha(1-\theta_A)}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}}$$

then

$$A(i_A) = \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} f(i_B/i_A)di_B = 1$$

b2) If

$$\frac{\frac{1}{2} - \alpha(1-\theta_A)}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}} \leq \frac{\theta_A - \frac{1}{2}}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}}$$

then

$$A(i_A) = \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}} f(i_B/i_A)di_B$$

b3) If

$$i_A \geq \frac{\theta_A - \frac{1}{2}}{1 - \frac{\alpha(1-\theta_A)}{\theta_A}}$$

then $A(i_A) = 0$. It then follows that E_ATB^F is given by

$$E_ATB^F = \int_0^{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_Bdi_A$$

$$+\int_{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\frac{\theta_A-\frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_Bdi_A$$

Similarly, E_BTB^F can be derived exactly the same way by replacing $TB(i_A)$ by $TB(i_B)$.

Social Surplus in the Case of Franchise (Not using the approval function).

1) $\theta_A \leq \frac{1}{2}$: ($A$ is a minority)

Following the same procedure as in the previous case, there are two subcases to be considered depending on the degree of preferences' conflict.

a) If $\alpha \leq \frac{1}{2(1-\theta_A)}$, then

$$E_A TB^F = \int_0^{\theta_A} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_B di_A$$

Proposals that are strongly opposed by A pass with a probability greater than zero since A is not very influential, but less than one since preferences are moderately opposed. Similarly, proposals that are strongly supported by A pass with a probability less than one since A is not very influential, but greater than zero since preferences are not very conflicting. Note that,

$$\frac{\partial E_A TB^F}{\partial \alpha} \leq 0$$

This simply says that as preferences become more conflicting, the expected payoff of A decreases.

b) If $\alpha \geq \frac{1}{2(1-\theta_A)}$, then

$$E_A TB^F = \int_0^{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_B di_A$$

$$+\int_{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\frac{\theta_A-\frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_A)f(i_A,i_B)di_B di_A$$

In this case, preferences are strongly opposed $\left(\alpha \geq \frac{1}{2(1-\theta_A)}\right)$. Since A is not influential $(\theta_A < \frac{1}{2})$, proposals that are strongly opposed by A $\left(i_A < \frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}\right)$ pass with probability one. Moderately supported proposals pass with a probability between zero and one. Strongly supported proposals $\left(i_A > \frac{\theta_A-\frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}\right)$ pass with zero probability. In particular, if $\alpha = 1$, a proposal which is supported by a majority of A will pass with zero probability. Also, proposals that are opposed by a majority of A will pass with probability one. Hence, it is not surprising that

$$\frac{\partial E_A TB^F}{\partial \alpha} \leq 0$$

For group B, the *ex ante* expected total surplus , $E_B TB^F$, is described below.

1) If $\theta_A \geq \frac{1}{2}$, then we have the following:

$$E_B TB^F = \int_{\frac{\theta_A - \frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B)f(i_A,i_B)di_B di_A +$$

$$\int_{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\theta_A} \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B)f(i_A,i_B)di_B di_A$$

where $f(i_A, i_B)$ is the joint p.d.f of (I_A, I_B) and $TB(i_B)$ is given by (10). Note that,

$$\frac{\partial E_B TB^F}{\partial \alpha} \leq 0$$

2) If $\theta_A \leq \frac{1}{2}$, there are two cases to be considered:

a)If $\alpha \leq \frac{1}{2(1-\theta_A)}$, then

$$E_B TB^F = \int_0^{\theta_A} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B)f(i_A,i_B)di_B di_A$$

Note that,

$$\frac{\partial E_B TB^F}{\partial \alpha} \leq 0$$

b) If $\alpha \geq \frac{1}{2(1-\theta_A)}$, then

$$E_B TB^F = \int_0^{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\alpha(1-\theta_A)+\alpha\frac{\theta_A-1}{\theta_A}i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B)f(i_A,i_B)di_B di_A$$

$$+ \int_{\frac{\frac{1}{2}-\alpha(1-\theta_A)}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}}^{\frac{\theta_A-\frac{1}{2}}{1-\frac{\alpha(1-\theta_A)}{\theta_A}}} \int_{\frac{1}{2}-i_A}^{1-\theta_A+\alpha\frac{\theta_A-1}{\theta_A}i_A} TB(i_B)f(i_A,i_B)di_B di_A$$

In this case, we have,

$$\frac{\partial E_B TB^F}{\partial \alpha} \geq 0$$

Theorem 1. *Suppose $C_0 < [E_A TB^{NF} - E_A TB^F][0]$ and that (25) holds. Then for $K \in (0, K^*)$, the subgame perfect equilibrium involves the franchise not being granted* *for any value of* $\alpha \in (0, \frac{1}{2(1-\theta_A)})$. *For* $K > K^*$, *we have alternating equilibria. For* $\alpha \in (0, \alpha_1)$, *the SPE involves no franchise. For* $\alpha \in [\alpha_1, \alpha_2)$, *the SPE involves franchise etc.*

Proof/

First, note that K^* guarantees that $C\left([E_BTB^F - E_BTB^{NF}][\alpha]\right)$ and $[E_ATB^{NF} - E_ATB^F][\alpha]$ cross at $\alpha = 1/2(1-\theta_A)$ since

$$K^*\left([E_BTB^F - E_BTB^{NF}][\frac{1}{2(1-\theta_A)}]\right) = [E_ATB^{NF} - E_ATB^F][\frac{1}{2(1-\theta_A)}]$$

To guarantee that this is the first crossing, the following inequality must hold for any $\alpha \leq 1/2(1-\theta_A)$.

$$K^*\left([E_BTB^F - E_BTB^{NF}][\alpha]\right) < [E_ATB^{NF} - E_ATB^F][\alpha]$$

It then follows that if (25) is satisfied, the first crossing of $[E_ATB^{NF} - E_ATB^F][\alpha]$ and $C\left([E_BTB^F - E_BTB^{NF}][\alpha]\right)$ occurs at $\alpha = \frac{1}{2(1-\theta_A)}$ for $K = K^*$. Hence, for $K \in (0, K^*)$, the l.h.s of (21) is strictly greater than the r.h.s for any $\alpha \leq 1/2(1-\theta_A)$. Hence threats are too costly for group B. Therefore the subgame perfect equilibrium outcome is one where the franchise is not granted for any value of α. If $K > K^*$, the l.h.s and the r.h.s of (21) may cross at $\alpha < 1/2(1-\theta_A)$. Hence the equilibrium will be dependent on α. If we denote the n^{th} crossing to the left of $1/2(1-\theta_A)$ by α_n, then we will have alternating equilibria. If $\alpha \in (0, \alpha_1)$, the SPE involves no franchise, for $\alpha \in [\alpha_1, \alpha_2)$, the SPE involves franchise, etc.

Theorem 2. *Suppose* $[E_ATB^{NF} - E_ATB^F][0] < C_0 < [E_ATB^{NF} - E_ATB^F][\frac{1}{2(1-\theta_A)}]$ *and that (26) holds. Then for* $K \in (0, K^*)$, *we have alternating equilibria. For* $\alpha \in (0, \alpha_1)$, *the subgame perfect equilibrium involves the franchise being granted. For* $\alpha \in [\alpha_1, \alpha_2)$, *the subgame perfect equilibrium involves no franchise etc. If* $K > K^*$, *the subgame perfect equilibrium involves the franchise being granted for any value of* α.

Proof/

If $C_0 \leq [E_A TB^{NF} - E_A TB^F][\frac{1}{2(1-\theta_A)}]$. If $K \in (0, K^*)$, then the first crossing occurs to the left of $\alpha = 1\big/2(1-\theta_A)$, and we have alternating equilibria. If $\alpha \in (0, \alpha_1)$, we have a threat equilibrium, if $\alpha \in [\alpha_1, \alpha_2)$, we have a no threat equilibrium, etc. If $K > K^*$, then the first crossing occurs to the right of $\alpha\big/2(1-\theta_A)$ and we have a threat equilibrium for any $\alpha \leq 1\big/2(1-\theta_A)$.

Theorem 3. *If $C_0 < [E_A TB^{NF} - E_A TB^F][0]$ and condition (27) holds. Then for $C_0 \in (0, C_0^*)$, the equilibrium outcome involves no franchise. For $C_0 \in (C_0^*, [E_A TB^{NF} - E_A TB^F][0])$, the equilibrium involves no franchise for $\alpha \in (0, \alpha_1)$ and franchise for $\alpha \in [\alpha_1, 1?big?2(1-\theta_A)]$. Moreover, if $C_0 > [E_A TB^{NF} - E_A TB^F][0]$, then the equilibrium involves the franchise being granted for any α.*

The proof is similar to the proofs of theorems 1 and 2.

References

Aumann, Robert J. and Kurz, Mordecai (1977): "Power and Taxes.," *Econometrica,* **45** pp. 1137-61.

Aumann, Robert J., Kurz, Mordecai and Neyman, Abraham (1983): "Voting for Public Goods.," *Rev. Econ. Studies,* **50** pp. 677-693.

Aumann, Robert J., Kurz, Mordecai, and Neyman, Abraham (1987): "Power and Public Goods.," *J. Econ. Theory,* **42** pp. 108-27.

Conley, John P. and Toossi, Ali (1999): "Evolution and Voting: How Nature Makes us Public Spirited.," Manuscript. UIUC Department of Economics.

Downs, Anthony (1957): *An Economic Theory of Democracy.* New York: Harper and Row.

Husted, Thomas A. and Kenny, Lawrence W. (1997): "The Effect of the Expansion of the Voting Franchise on the Size of the Government," *J.P.E.,* **105** pp. 54-82.

Kenny, Lawrence W. (1978): "The Collective Allocation of Commodities in a Democratic Society: A Generalization.," *Public Choice,* **33** pp. 117-120.

Ledyard, John O. (1981): "The Paradox of Voting and Candidate Competition: A General Equilibrium Analysis," in *Essays in Contemporary Fields of Economics,* by Hornwich, G. and Quirk, J. eds..

Purdue University Press/

Ledyard, John O. (1984): "The Pure Theory of Large Two-Candidate Elections," *Public Choice* , **44** pp. 7-41.

Meltzer, Allan H. and Richard, Scott F. (1981): "A Rational Theory of the Size of Government," *J.P.E.,* **89** pp. 914-27.

Mueller, Dennis C. (1989): *Public Choice II.* Cambridge University Press.

Palfrey, Thomas R. and Rosenthal, Howard (1983): "A Strategic Calculus of Voting.," *Public Choice* , **41** pp. 7-53.

Palfrey, Thomas R. and Rosenthal, Howard (1985): "Voter Participation and Strategic Uncertainty.," *American Polit. Sci. Rev.,* **79** pp. 62-78.

Temimi, Akram. (1996): "Essays in Cooperative Game Theory and Public Finance," Ph.D. dissertation, University of Illinois.

Uhlaner, Carole J. (1989a): "Rational Turnout: The Neglected Role of Groups," *American J. Polit. Sci.,* **33** pp. 390-422.

Uhlaner, Carole J. (1989b): "Relational Goods and Participation: Incorporating Sociability into a Theory of Rational Action," *Public Choice* , **62** pp. 253-85.

Uhlaner, Carole J. (1993): "What the Downsian Voter Weighs: A Reassessment of the Costs and Benefits of Action.," in *Information, Participation, and Choice: An Economic Theory of Democracy in Perspective,,* by Groffman, B. ed., Ann Arbor MI: University of Michigan Press.

Wooders, Myrna H. (1994): "A Remark on Aumann, Kurz and Neyman (1987)," Manuscript. Toronto, University of Toronto, Department of Economics.

Captions for Figures

Figure 1- Benefit functions for groups A and B from a particular proposal.

Figure 2- The proposal represented by the dotted line generates losses to every member from group A while it generates positive benefits to every member of group B. The proposal represented by the solid line is the exact opposite.

Figure 3- Extensive form game between groups A and B.

Figure 4- Under constant returns to scale threat technology, the equilibrium alternates between franchise and no franchise as the degree of preference conflict increases.

Figure 5- Under increasing returns to scale, the equilibrium involves no franchise when the degree of conflict in preferences is low and enfranchisement when it is high.